

(Dis-)Aggregate Consumption & Monetary Policy

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18.02.2019

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Motivation

Q: How does Consumption Behavior Influence Monetary Policy?

Standard New Keynesian Model:

- Sticky Prices
⇒ Monetary policy has real effects
- Bundle of Goods
+ CES preferences ⇒ aggregate consumption
- No Unemployment

But:

- Goods differ in *price flexibility* and *consumption elasticity*
- Unemployment does exist

Empirical Findings

Intensive Margin Consumption:

- ΔC_t due to permanent income change other than employment status (wages, hours, ...)
- Households consume disproportionately more **sticky price goods** (luxuries)

Extensive Margin Consumption:

- ΔC_t due to change in employment
- Households consume disproportionately more **flexible price goods** (necessities)

Expansionary Monetary Policy:

- no significant effect on weekly earnings
- **increases employment**

Theoretical Findings

Textbook New Keynesian model with:

- Search and Matching Friction \Rightarrow Unemployment
- **Non-homothetic preferences** (Necessity vs. Luxury consumption)
- **Home-Production** of Necessity
- Representative Family \Rightarrow Complete Consumption Insurance

Comparison to Standard NK Model:

- Worse Trade-Off - Real Output vs. Inflation
- State-Dependence
- Optimal Policy reacts to Unemployment

- **Consumption Response to Income Changes**

Expected vs Unexpected: Johnson et al (2006), Parker et al (2013)

Transitory vs Permanent: Blundell et al (2008), Arellano et al (2017)

Intensive vs Extensive: Alonso (2016), Hubner (2018)

- **Home Production & Shopping Time Behavior**

Benhabib et al (1991), Aguiar, Hurst (2005, 2007, 2013), Nevo, Wong (2015)

Subset of Goods

- **New Keynesian Models**

Multi-sector: Barsky et al (2006), DiPace, Hertweck (2016)

Search-and-Matching: Trigari (2009), Blanchard, Gali (2010), Ravenna, Walsh (2012)

Non-homotheticity: Huang et al (2013), Cavallari (2018)

Home-Production: Gnocchi et al (2016)

Outline

- 1 Introduction
- 2 Empirical Findings
 - ▶ Intensive Margin Consumption
 - ▶ Extensive Margin Consumption
 - ▶ Monetary Policy
- 3 Theoretical Findings
 - ▶ Comparison to New Keynesian Model
 - ▶ State-Dependence
 - ▶ Welfare Analysis

Data - Consumption Expenditures

CEX - Consumption Expenditure Survey

- US *household level* consumption expenditures
- Quarterly 1980-2016
- Households (HH) interviewed for 4 consecutive quarters
- 22 Consumption Categories [Details](#)
 - ▶ E.g. Apparel, Gasoline, Recreation

Data - Price Flexibility

1 Price Change Frequency

- ▶ Micro Price Data underlying the Consumer Price Index (CPI)
- ▶ Average, regular price change frequencies 1988-2005
 - ★ Klenow, Kryvtsov (2008) - 1988-1997
 - ★ Nakamura, Steinsson (2008) - 1998-2005

2 Price Cyclicity $\log \frac{P_{ct}}{P_t} = \alpha + \beta_c \log Y_t$

- ▶ $\log \frac{P_{ct}}{P_t} =$ (hp-filtered) log relative price index (NIPA)
- ▶ $\log Y_t =$ (hp-filtered) log real GDP

Intensive Margin Consumption

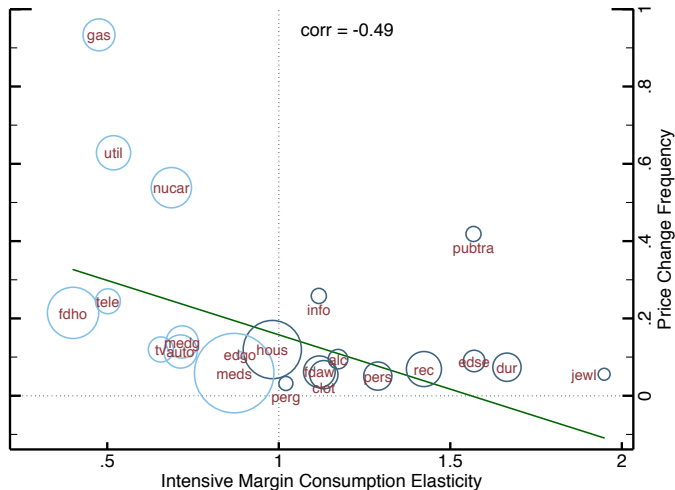
- Income Elasticities (Engel Curves)
- Sample: No HH member changes employment status
~ 45000 Households
- Specification:

$$\tilde{y}_{cht} = \alpha_{ct} + \alpha_{ch} + \beta_c^i \cdot \log C_{ht} + \gamma_c \cdot X_{ht} + \nu_{cht}$$

- ▶ $\tilde{y}_{cht} = \frac{y_{cht}}{y_{ct}}$ - zero expenditures
 - ▶ $\log C_{ht}$ - proxy for permanent income
 - ▶ β_c^i - expenditure elasticity of good c
- Measurement Error in y_{cht}
 - ▶ Add expenditures from 2nd-4th interview
 - ▶ Instrument by expenditures from 1st interview

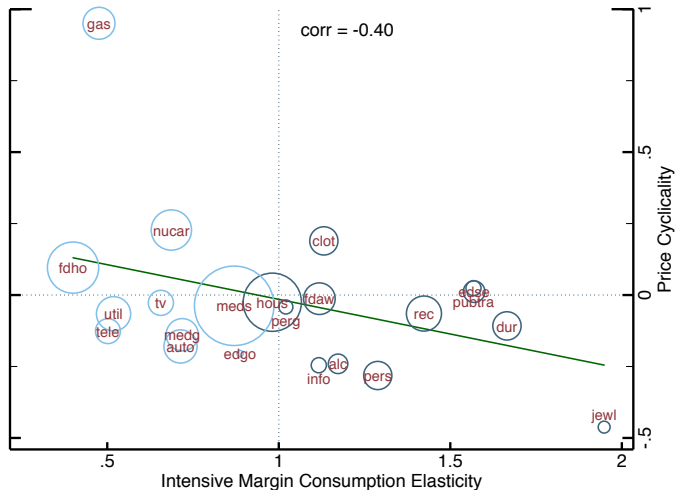
Intensive Margin Consumption

- HHs spend **intensive margin** income increases on **sticky price goods**



Intensive Margin Consumption

- HHs spend **intensive margin** income increases on **sticky price goods**



Extensive Margin Consumption

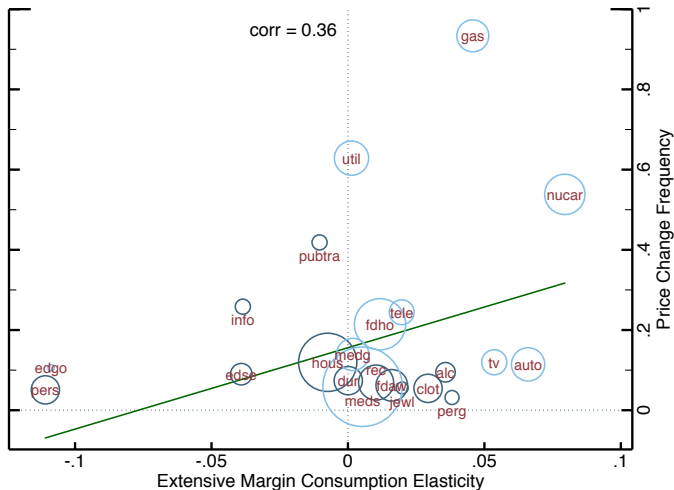
- Sample: Households with change in employment status
~ 15000 Households
- Specification:

$$\tilde{y}_{cht} = \alpha_{ct} + \alpha_{ch} + \beta_c^e \cdot \#Earnings_{ht} + \gamma_c \cdot X_{ht} + \nu_{cht}$$

- ▶ $\tilde{y}_{cht} = \frac{y_{cht}}{\bar{y}_{ct}}$ - zero expenditures
- ▶ $\#Earnings_{ht}$ - Number of Earners in HH h at t
- ▶ β_c^e - Semi-elasticity of expenditure on good c

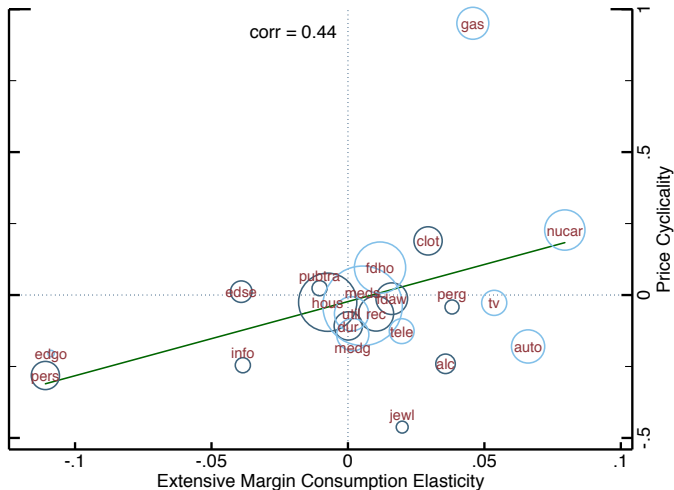
Extensive Margin Consumption

- HHs spend **extensive margin** income increases on **flexible price goods**



Extensive Margin Consumption

- HHs spend **extensive margin** income increases on **flexible price goods**

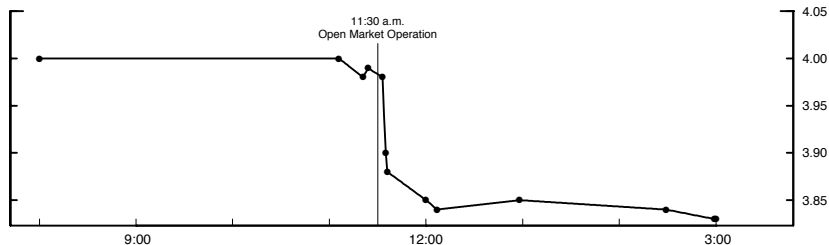


Data - Monetary Policy Shocks

Monetary Shocks

- High-frequency changes of Federal Funds Futures 1990-2007 [Details](#)

April 9, 1992 (April 1992 contract)



Gurkaynak et al (2005)

- Sum to quarterly measure ε_t^M

Data - Labor Market

CPS - Current Population Survey

- Monthly 4-8-4 Sampling Scheme
- Age-Education-Gender Cohorts
- Quarterly 1990-2007
- Specification:

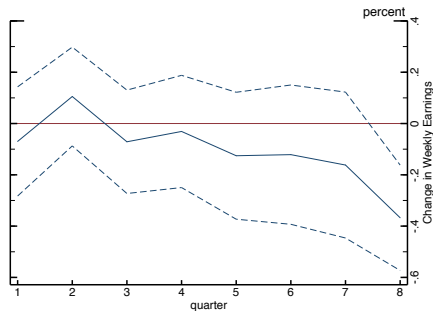
$$\Delta y_{it} = \alpha_i + \sum_{k=1}^K \beta_k \cdot \varepsilon_{t-k}^- + \sum_{k=1}^K \gamma_k \cdot \varepsilon_{t-k}^+ + \nu_{it}$$

- ▶ $\Delta y_{it} \in \{\text{real log weekly earnings, employment rate}\}$
- ▶ α_i - Cohort FE
- ▶ $\varepsilon_t^- = \min\{\varepsilon_t^M, 0\}$ - expansionary shock
- ▶ $\varepsilon_t^+ = \max\{0, \varepsilon_t^M\}$ - contractionary shock

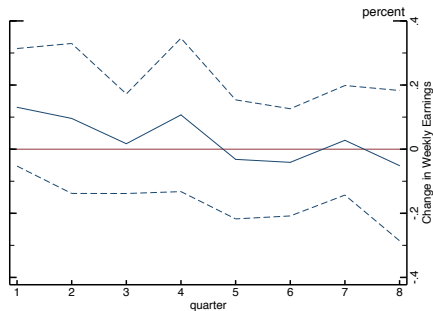
Impulse Responses to MP - Real Weekly Earnings

- Monetary policy does not affect the intensive margin

a) contractionary



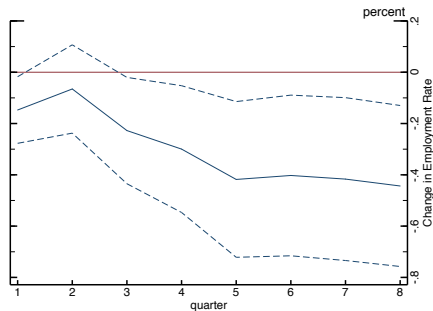
b) expansionary



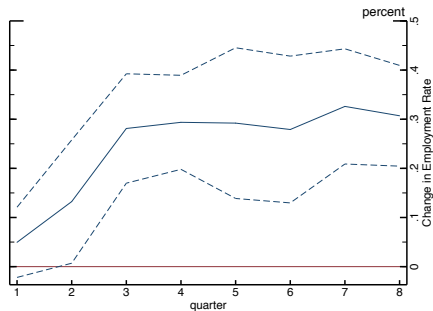
Impulse Responses to MP - Employment

- Expansionary monetary policy increases employment

a) contractionary



b) expansionary



Quick Recap

- 1 **Intensive margin** income increases are spend on **sticky price** goods
- 2 **Extensive margin** income increases are spend on **flexible price** goods
- 3 Monetary policy works through the **extensive margin**

(Textbook) New Keynesian Model

Households

- Representative Agent
 - ▶ Unemployed & Employed members
 - ▶ Complete Consumption Insurance
- CES preferences
 - ▶ Bundle of Goods \Rightarrow Aggregate Consumption

Firms

- Retailer
 - ▶ Bundles intermediate goods
- Intermediate
 - ▶ Monopolistic Competition + Calvo Sticky Prices θ
 - ▶ Production
- Intermediate
 - ▶ Production + Hiring
 - ▶ Perfect Competition

Extended New Keynesian Model

Households

- Representative *Family*
 - ▶ Unemployed & Employed members
 - ▶ Complete Consumption Insurance
- Non-homothetic preferences
 - ▶ Necessity vs. Luxury Consumption
- Home Production of Necessity

Firms

- Retailer $s \in \{N, L\}$
 - ▶ Bundles wholesale goods
- Wholesale $s \in \{N, L\}$
 - ▶ Monopolistic Competition + Calvo Sticky Prices θ_s
 - ▶ Production
- Intermediate
 - ▶ Production + Hiring
 - ▶ Perfect Competition

Extended New Keynesian Model

Households

- Representative *Family*
- **Non-homothetic preferences** over bundle of goods
- **Home Production** of Necessity

Firms

- Retailer $s \in \{N, L\}$
- Wholesale $s \in \{N, L\}$
- Intermediate

Labor Market

- Members \Leftrightarrow Intermediates
- Nash Bargained Wages

Household Preferences

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{V(C_{Nt}, C_{Lt})^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\varphi}}{1+\varphi} \right)$$

where

$$V(C_{Nt}, C_{Lt}) = \left(C_{Nt}^{1-\lambda} + \eta \frac{1-\lambda}{1-\phi} C_{Lt}^{1-\phi} \right)^{1/(1-\lambda)}$$

and

$$C_{Nt} = (X_{Nt}^\rho + \psi(1 - N_t)^\rho)^{1/\rho}$$

Household Constraints

Budget Constraint

$$P_{Nt}X_{Nt} + P_{Lt}C_{Lt} + Q_tB_t = W_tN_t + B_{t-1} + \Phi_t + T_t$$

Evolution of Employment:

$$N_t = (1 - \delta)N_{t-1} + H_t$$

where

$$H_t = x_t U_t^0$$

$$U_t^0 = 1 - N_{t-1} + \delta N_{t-1} = 1 - (1 - \delta)N_{t-1}$$

Firms

Retail Firm $s \in \{N, L\}$

$$Y_{st}(j) = \left(\frac{P_{st}(j)}{P_{st}} \right)^\epsilon Y_{st}$$

Wholesale Firms $s \in \{N, L\}$

$$\max_{P_{st}(j)} E_t \sum_{k=0}^{\infty} \theta_s^k Q_{t,t+k} Y_{st,t+k}(j) [P_{st}(j) - P'_{t+k}]$$

Optimal Price

$$P_{st}^* = \frac{\epsilon}{\epsilon - 1} E_t \left\{ \frac{\sum_{k=0}^{\infty} \theta_s^k Q_{t,t+k} Y_{st,t+k} P_{st+k}^\epsilon P'_{t+k}}{\sum_{k=0}^{\infty} \theta_s^k Q_{t,t+k} Y_{st,t+k} P_{st+k}^\epsilon} \right\}$$

Sectoral Price

$$P_{st} = [\theta_s P_{st-1}^{1-\epsilon} + (1 - \theta_s) P_{st}^{*1-\epsilon}]^{\frac{1}{1-\epsilon}}.$$

Intermediate Firm

$$\max_{\{N_t\}} E_t \left\{ \sum_{k=0}^{\infty} Q_{t,t+k} (P'_{t+k} Y'_{t+k} - W_{t+k} N_{t+k} - P_{N_{t+k}} G_{t+k} H_{t+k}) \right\}$$

s.t.

$$Y'_t = A_t N_t^{1-\alpha}$$

$$N_t = (1 - \delta) N_{t-1} + H_t$$

where

$$G_t = G(x_t) = \Gamma x_t^\gamma$$

$$\ln A_t = \rho_a \ln A_{t-1} + \varepsilon_t^a \quad \varepsilon \sim N(0, \sigma_a^2)$$

FOC:

$$\begin{aligned} MRPN_t &= P'_t (1 - \alpha) A_t N_t^{-\alpha} \\ &= W_t + P_{N_t} G_t - (1 - \delta) E_t \{ Q_{t,t+1} P_{N_{t+1}} G_{t+1} \} \end{aligned}$$

Wage Bargaining

Nash Bargaining

$$\max_{W_t} (S_t^H)^{1-\xi} (S_t^F)^\xi$$

s.t.

$$S_t^H = W_t - MRS_t + (1 - \delta) E_t \{ Q_{t,t+1} S_{t+1}^H \}$$

$$S_t^F = MRPN_t - W_t + (1 - \delta) E_t \{ Q_{t,t+1} S_{t+1}^F \}$$

where

$$MRS_t = \frac{P_{Lt}}{\eta C_{Lt}^\phi} \left[\chi N_t^\varphi V(C_{Nt}, C_{Lt})^{\sigma-\lambda} + C_{Nt}^{-\lambda} \psi \left(\frac{1-N_t}{C_{Nt}} \right)^{\rho-1} \right]$$

FOC:

$$W_t = \xi MRS_t + (1 - \xi) MRPN_t.$$

$$P_{Nt} G_t - (1 - \delta) E_t \{ Q_{t,t+1} P_{Nt+1} G_{t+1} \} = \xi (MRPN_t - MRS_t)$$

Policy

Fiscal Policy

$$T_t + Q_t B_t = B_{t-1}$$

Monetary Policy

$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\Pi_t}{\Pi} \right)^{\phi_p} \left(\frac{Y_t}{Y} \right)^{\phi_y} \right]^{1-\rho_r} \cdot \varepsilon_t^r$$

where

$$\Pi_t = P_t / P_{t-1}$$

$$\ln \varepsilon_t^r \sim N(0, \sigma_\varepsilon^2)$$

and

$$P_t = \frac{\text{Nominal GDP}}{\text{Real GDP}} = \frac{P_{Nt} Y_{Nt} + P_{Lt} Y_{Lt}}{P_N Y_{Nt} + P_L Y_{Lt}}$$

Market Clearing

Final Goods

$$Y_{Nt} = X_{Nt} + H_t G_t$$

$$Y_{Lt} = C_{Lt}$$

Intermediate Goods

$$\int_0^1 M_{Nt}(i) di + \int_0^1 M_{Lt}(i) di = Y_t'$$

$$S_{Nt} Y_{Nt} + S_{Lt} Y_{Lt} = A_t N_t^{1-\alpha}$$

where

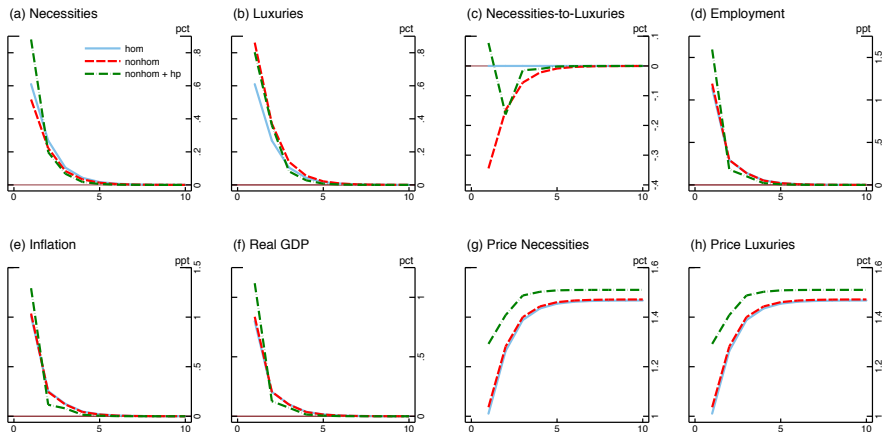
$$S_{st} \equiv \int_0^1 \left(\frac{P_{st}(i)}{P_{st}} \right)^{-\epsilon} di \quad s \in \{N, L\}$$

Calibration

Parameter	Value	Description
β	0.99	Discount Factor
σ	1	Intertemporal EoS
φ	5	(Inverse) Frisch Elasticity
χ	0.39	Disutility of Employment
λ	1.01	Relative Price Elasticity = 1.7
ϕ	0.36	Relative Expenditure Elasticity = 0.66
η	1.04	Expenditure Share Necessity = 0.65
ψ	0.15	Consumption weight on Home Production
ρ	2/3	EoS btw. Necessities and Home Production
ϵ	6	Demand Elasticity
θ_N	0.69	Price Change Frequency Necessities
θ_L	0.83	Price Change Frequency Luxuries
α	0.33	Labor Income Share
δ	0.1228	Separation Rate
γ	1	Matching Function Equivalence
ξ	0.5	Bargaining Power
Γ	0.1183	Hiring Cost
ρ_r	0.9	Monetary Policy Inertia
ϕ_p	1.5	Inflation Response
ϕ_y	0.5/4	Output Response
σ_ϵ^2	0.0025	Monetary Policy Volatility
ρ_a	0.9	Productivity Persistence
σ_a^2	0.008	Productivity Volatility

IRF MP for $\theta_N = \theta_L$

- Home-Production introduces an *additional demand channel*



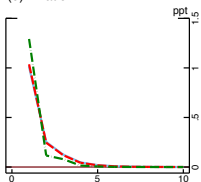
IRF MP for $\theta_N \neq \theta_L$

- Additional demand occurs for flexible price goods

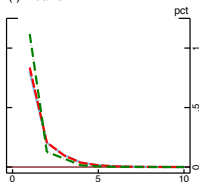
a) $\theta_N = \theta_L$

b) $\theta_N \neq \theta_L$

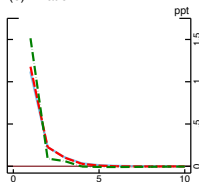
(e) Inflation



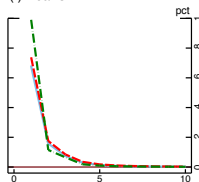
(f) Real GDP



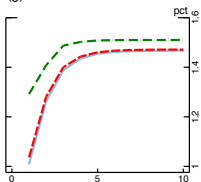
(e) Inflation



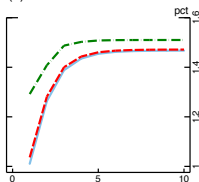
(f) Real GDP



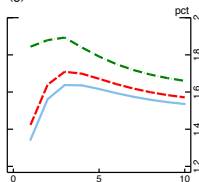
(g) Price Necessities



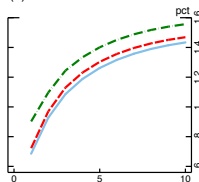
(h) Price Luxuries



(g) Price Necessities

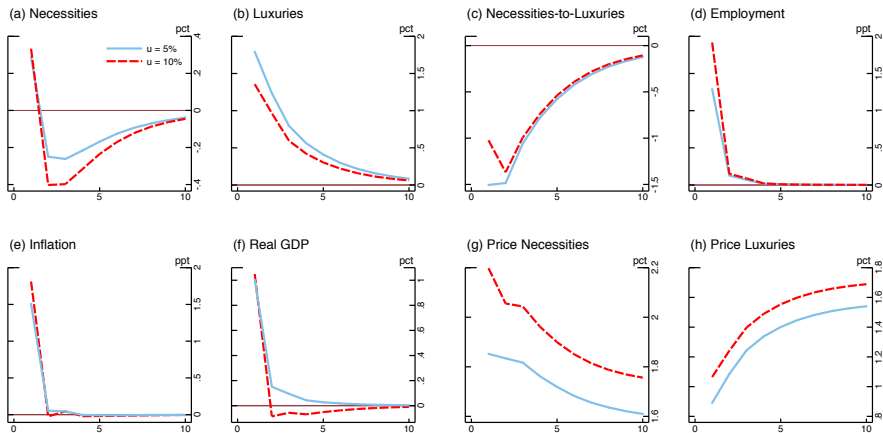


(h) Price Luxuries



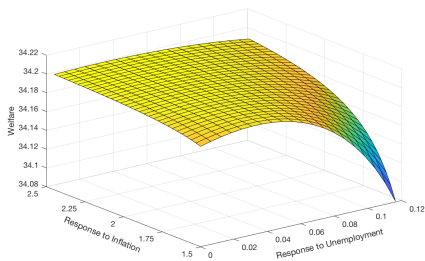
State-Dependence of MP

- Higher unemployment mainly leads to increased inflation



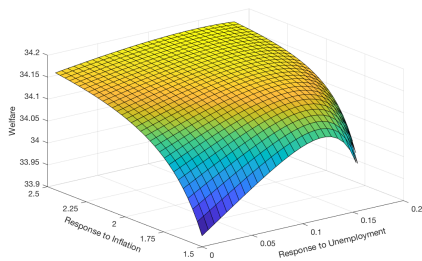
$$\frac{R_t}{R} = \left(\frac{R_{t-1}}{R} \right)^{\rho_r} \left[\left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_p} \left(\frac{Y_t}{\bar{Y}} \right)^{\phi_y} \left(\frac{U_t}{\bar{U}} \right)^{\phi_u} \right]^{1-\rho_r} \varepsilon_t^r$$

a) Optimal Simple Rule



$$\phi_u^* = 0.037$$

b) $\phi_y = 0.125$



$$\phi_u^* = 0.153$$

Conclusion

- 1 Consumption Behavior differs at the **intensive** and **extensive** margin
- 2 Monetary Policy works through the **extensive** margin
- 3 Monetary Policy should take unemployment into account ...
... even if inflation is the main concern

Consumption Categories

- Apparel
- Jewelry
- Housing
- Utilities
- Durables
- New & Used Cars
- Gasoline
- Car Maintenance
- Public Transport
- Educational Goods
- Educational Services
- Telephone Services
- Information
- Medical Goods
- Medical Services
- TV & Audio
- Recreation
- Personal Goods
- Personal Services
- Food at Home
- Food Away
- Alcohol

Monetary Policy Shocks

MP shock

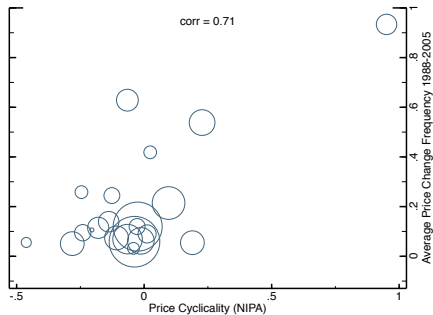
$$\epsilon_t^M = \frac{D}{D-T} (ffr_{t+\Delta^+} - ffr_{t-\Delta^-})$$

where

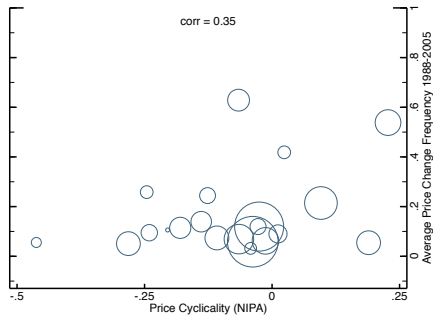
- t - FOMC announcement date
- ffr - Federal Funds Futures Rate ...
- ... $\Delta^+ = 45\text{min after}$ / $\Delta^- = 15\text{min before}$ FOMC Announcement
- $\frac{D}{D-T}$ - Adjustment Term
- D - Number of Days in FOMC announcement month

Correlation of Price Flexibility Measures

a) NIPA

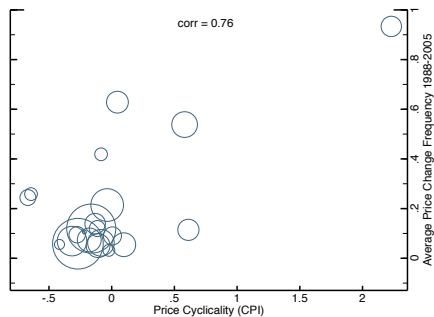


b) NIPA - no gasoline

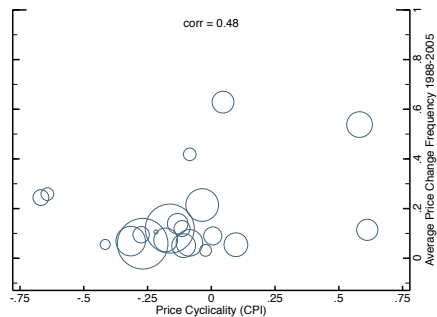


Correlation of Price Flexibility Measures

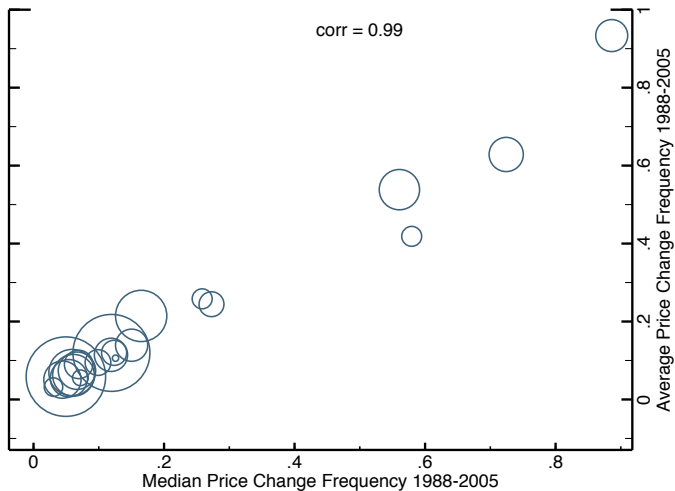
c) CPI



d) CPI - no gasoline



Correlation of Price Flexibility Measures



back

Robustness - Price Change Frequency

$$\beta_c^i = \alpha + \gamma \cdot \text{Price Flexibility}_c + \varepsilon$$

a) Average Price Change Frequency 1988-2005

	baseline (1)	male heads (2)	age 20-65 (3)	CEX share (4)	EC deflator (5)	log (6)
beta	-.837*** [.200]	-.815*** [.203]	-.848*** [.196]	-.810*** [.179]	-.745*** [.198]	-1.09** [.426]
correlation	-.485	-.474	-0.494	-.523	-.453	-.408

b) Median Price Change Frequency 1988-2005

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	-.731*** [.183]	-.706*** [.195]	-.743*** [.177]	-.711*** [.154]	-.645*** [.185]	-.968*** [.331]
correlation	-.445	-.430	-.454	-.478	-.412	-.379

Robustness - Price Cyclicity

$$\beta_c^i = \alpha + \gamma \cdot \text{Price Flexibility}_c + \varepsilon$$

c) Price Cyclicity (NIPA) 1980-2016

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	-.666*** [.210]	-.665*** [.209]	-.673*** [.214]	-.555*** [.174]	-.625*** [.208]	-.664* [.373]
correlation	-.402	-.402	-.408	-.377	-.396	-.256

d) Price Cyclicity (CPI) 1980-2016

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	-.249*** [0.070]	-.246*** [.070]	-.249*** [.067]	-.212*** [.059]	-.225*** [.068]	-.243 [.147]
correlation	-.345	-.341	.346	-.345	-.327	-.217

Robustness - Price Change Frequency

$$\beta_c^e = \alpha + \gamma \cdot \text{Price Flexibility}_c + \varepsilon$$

a) Average Price Change Frequency 1988-2005

	baseline (1)	male heads (2)	age 20-65 (3)	CEX share (4)	EC deflator (5)	log (6)
beta	.064* [.033]	.058* [.030]	.047* [.023]	.046* [.024]	.064* [.033]	.021 [.037]
correlation	.359	.354	.325	.359	.359	.121

b) Median Price Change Frequency 1988-2005

	baseline (1)	male heads (2)	age 20-65 (4)	CEX shares (4)	EC deflator (5)	log (6)
beta	.058* [.033]	.054* [.029]	.044* [.024]	.041 [.026]	.058* [.033]	.021 [.037]
correlation	.346	.347	.317	.335	.346	.123

Robustness - Price Cyclicity

$$\beta_c^e = \alpha + \gamma \cdot \text{Price Flexibility}_c + \varepsilon$$

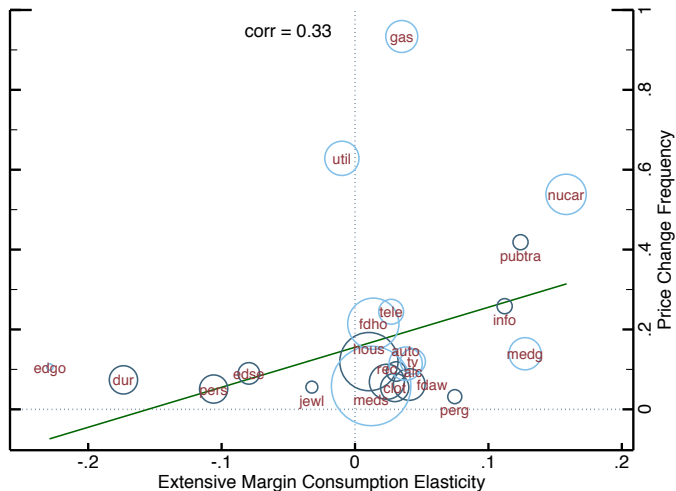
c) Price Cyclicity (NIPA) 1980-2016

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	.075* [.042]	0.060 [.035]	.056** [.027]	.046** [.019]	.075* [.042]	.028 [.020]
correlation	.441	.380	.401	.377	.441	.172

d) Price Cyclicity (CPI) 1980-2016

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	.031*** [.014]	.027* [.014]	.026** [.011]	.026** [.010]	.031** [.014]	.023* [.013]
correlation	.415	0.390	.429	.513	.415	.316

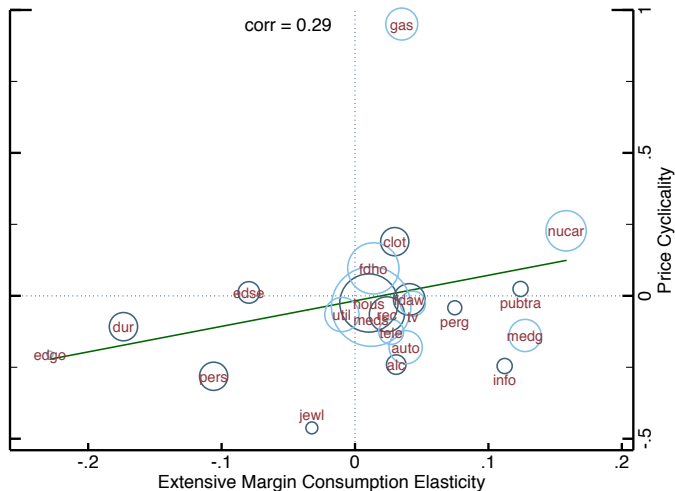
Robustness - Employment Status of HH Head



back

Robustness

Robustness - Employment Status of HH Head



back

Robustness

Robustness - Employment Status of HH Head

$$\beta_c^e = \alpha + \gamma \cdot \text{Price Flexibility}_c + \varepsilon$$

a) Average Price Change Frequency 1988-2005

	baseline (1)	male heads (2)	age 20-65 (3)	CEX share (4)	EC deflator (5)	log (6)
beta	.107 [.076]	.129* [.071]	.100 [.075]	.066 [.052]	.107 [.076]	.014 [.058]
correlation	.328	.318	.348	.267	.328	.057

b) Median Price Change Frequency 1988-2005

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	.108 [.074]	.132* [.069]	.101 [.072]	.066 [.054]	.108 [.074]	.013 [.057]
correlation	.345	.342	.367	.275	.345	.054

Robustness - Employment Status of HH Head

$$\beta_c^e = \alpha + \gamma \cdot \text{Price Flexibility}_c + \varepsilon$$

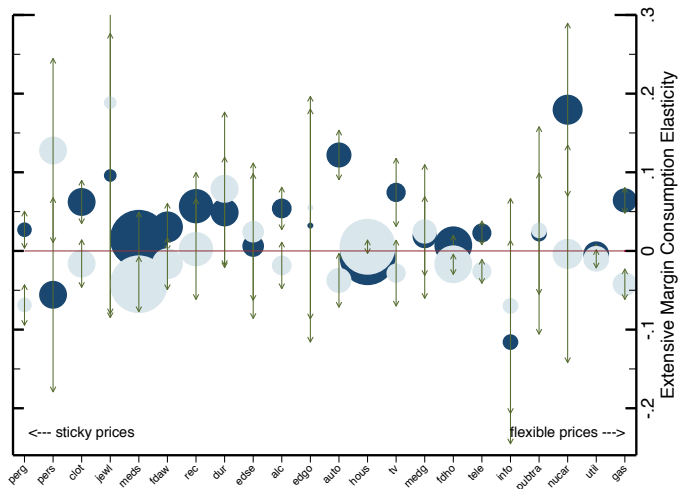
c) Price Cyclicity (NIPA) 1980-2016

	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	.089 [.069]	.166 [.116]	.053 [.049]	.058 [.041]	.089 [.069]	.046 [.028]
correlation	.281	.426	.189	.245	.281	.193

d) Price Cyclicity (CPI) 1980-2016

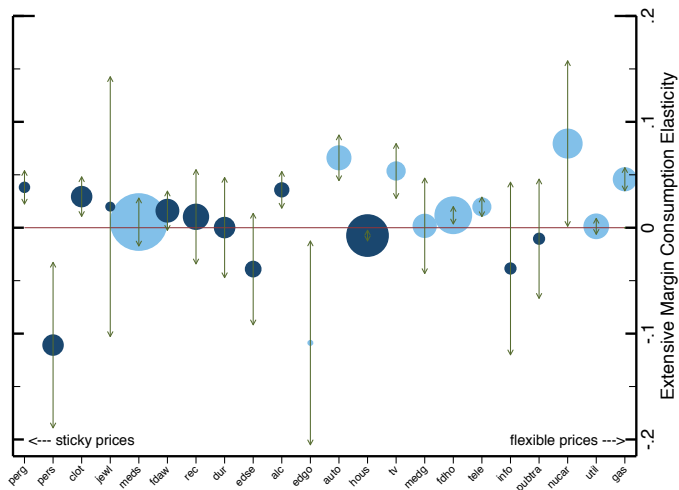
	baseline (1)	male heads (2)	age 20-65 (3)	CEX shares (4)	EC deflator (5)	log (6)
beta	.029 [.024]	.036* [.021]	.023 [.026]	.022 [.015]	.029 [.024]	0.030** [.014]
correlation	.209	.211	.192	.221	.209	.299

Robustness - Symmetry



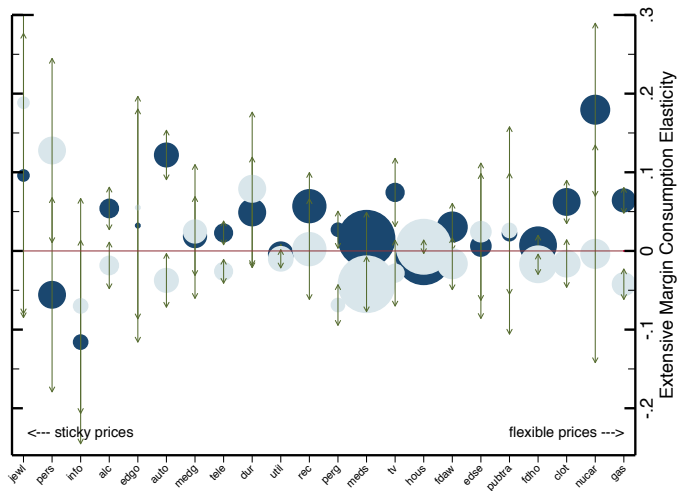
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Robustness - Significance



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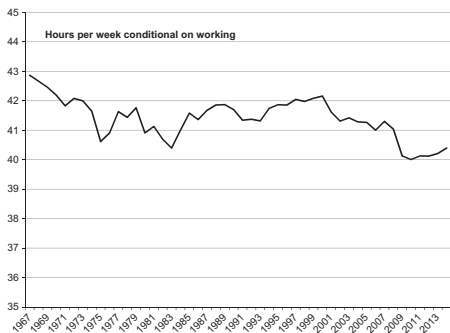
Robustness - Symmetry



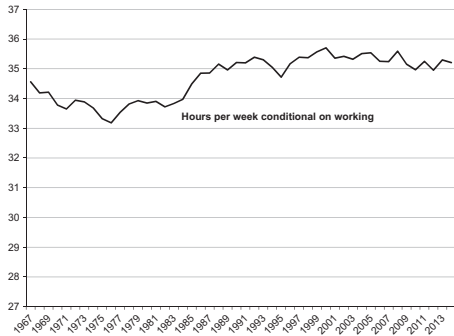
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Hours worked per week

a) men



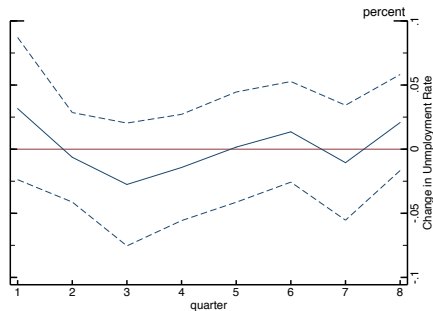
b) women



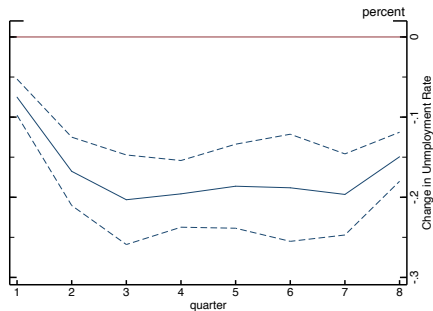
Impulse Responses to MP - Unemployment

- Expansionary monetary policy decreases unemployment

a) contractionary

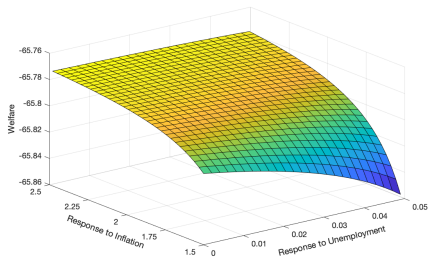


b) expansionary



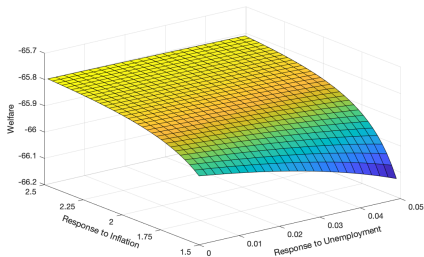
Welfare in NK model

a) Optimal Simple Rule



$$\phi_u^* = 0$$

b) $\phi_y = 0.125$



$$\phi_u^* = 0$$

Equalization of Wages

Intermediate Firm FOC:

$$\begin{aligned}MRPN_t(i) &= P_t^I(1 - \alpha)A_tN_t(i)^{-\alpha} \\ &= W_t(i) + P_{Nt}G_t - (1 - \delta)E_t \{Q_{t,t+1}P_{Nt+1}G_{t+1}\}\end{aligned}$$

Nash Bargaining FOC:

$$W_t(i) = \xi MRS_t + (1 - \xi)MRPN_t(i).$$

where

$$MRS_t = \frac{P_{Lt}}{\eta C_{Lt}^\phi} \left[\chi N_t^\varphi V(C_{Bt}, C_{Lt})^{\sigma - \lambda} + C_{Bt}^{-\lambda} \psi \left(\frac{1 - N_t}{C_{Bt}} \right)^{\rho - 1} \right]$$