Monetary-Fiscal Forward Guidance

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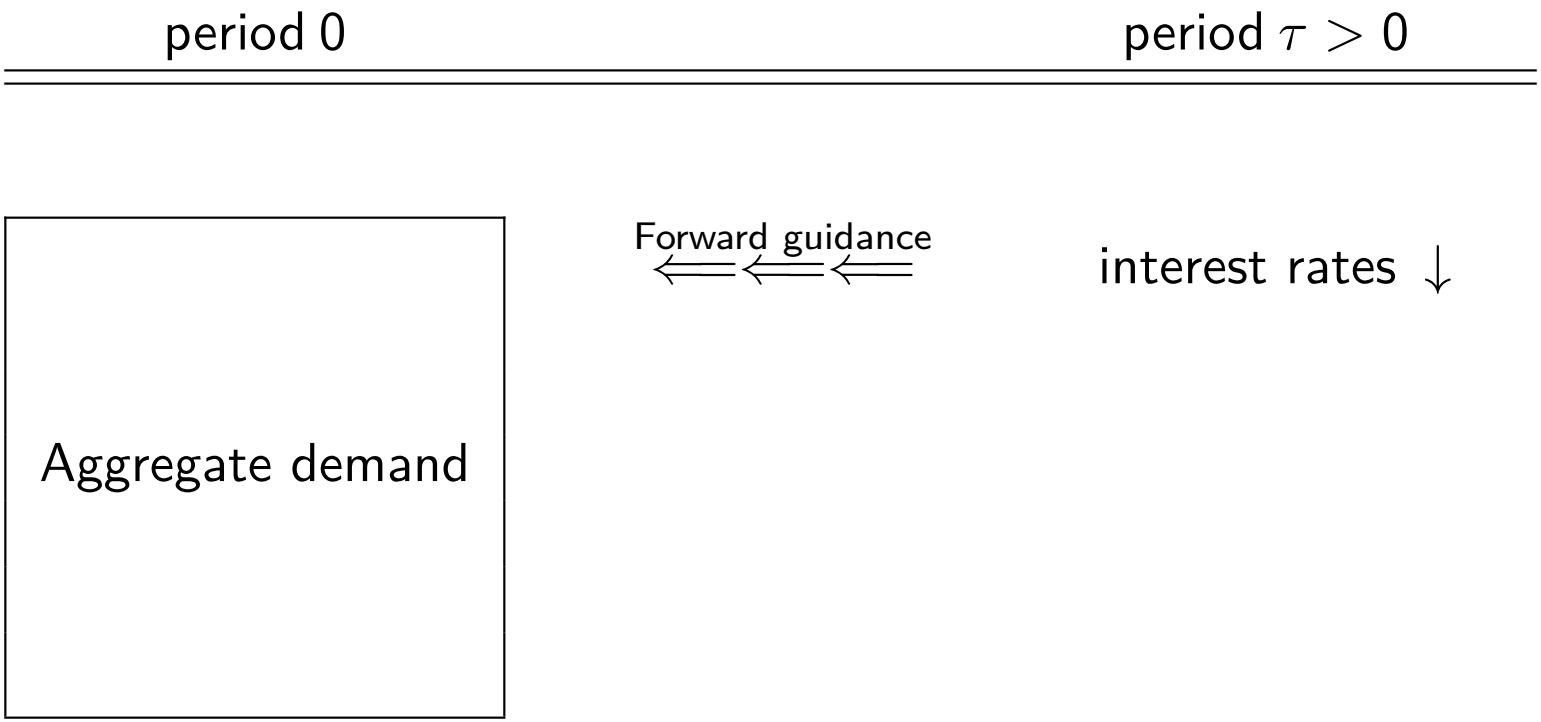
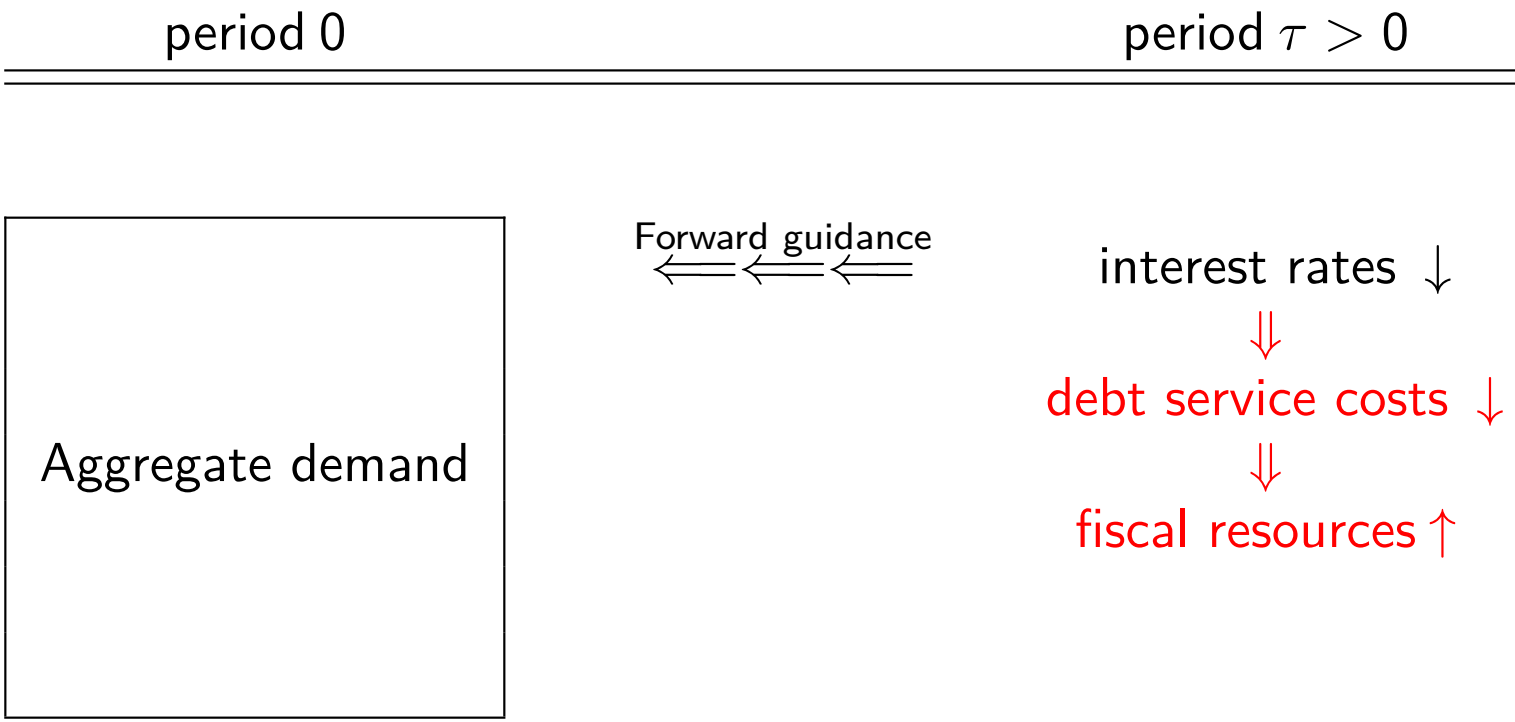
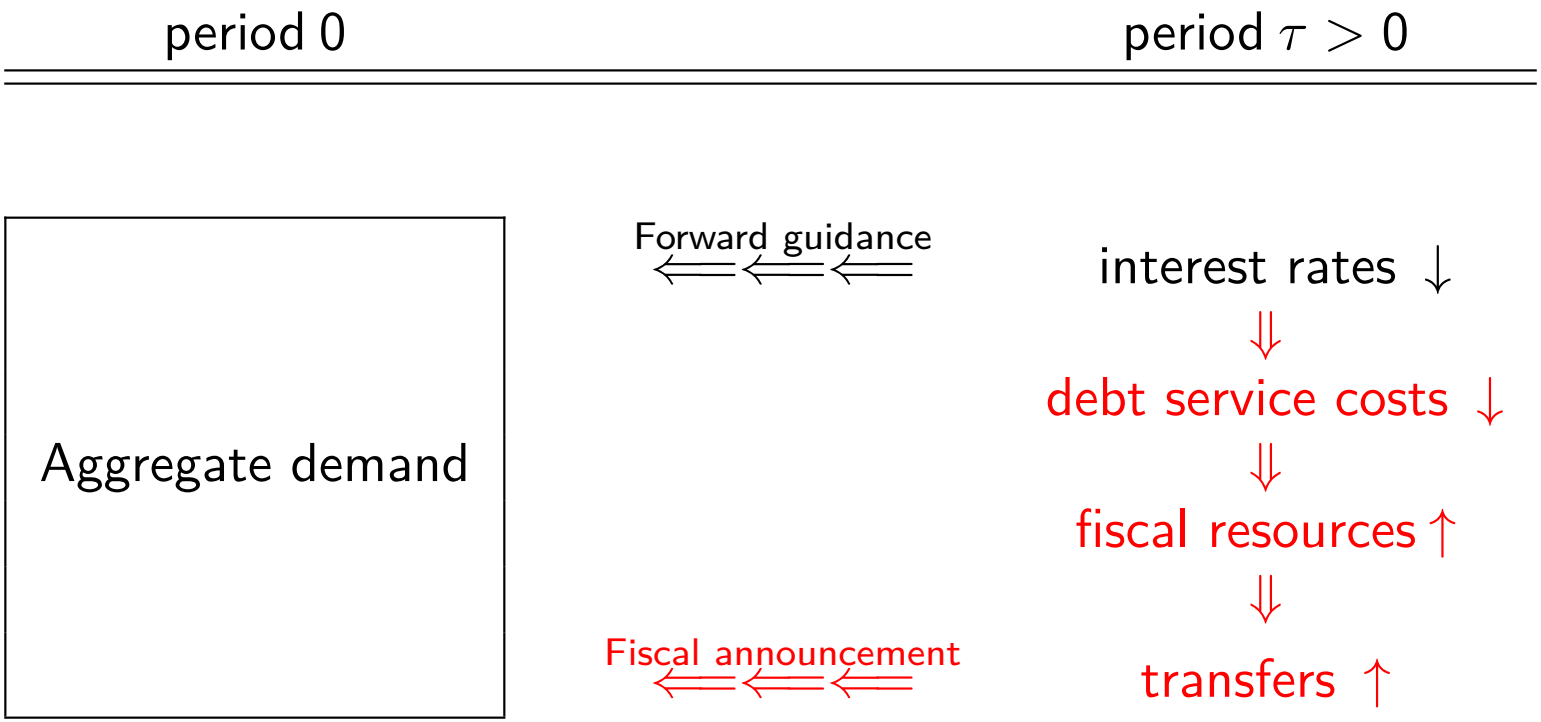
Forward guidance (Odyssean)



Forward guidance and a simple monetary-fiscal interaction



Monetary-fiscal forward guidance (MFFG)



MFFG and constrained monetary/fiscal policies

The announced fiscal stimulus:

- neutral for the path of public debt
- neutral for the path of taxes
- increasing in the level of public debt

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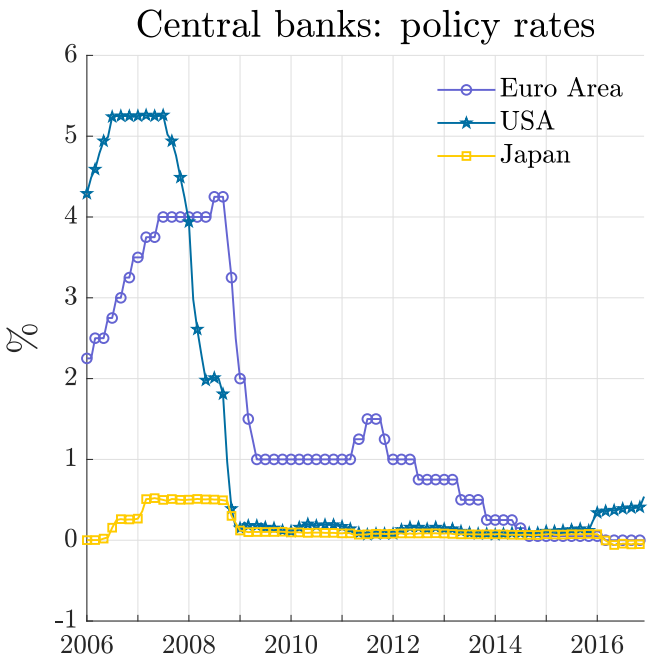

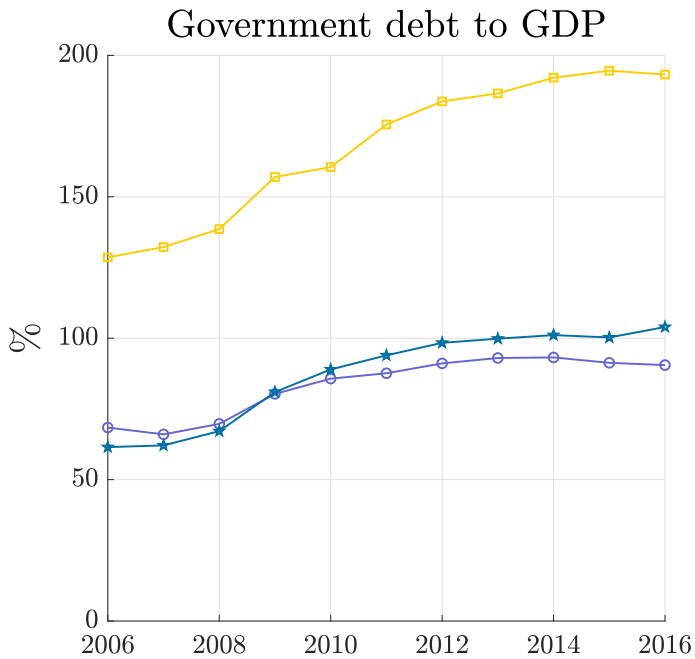

MFFG and (un)bounded rationality

Rational expectations:

- Ricardian agents aware of monetary-fiscal interactions
- no additional information conveyed by the fiscal announcement
- the effects of MFFG and FG are **the same**

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Bounded rationality:

- confirmed by laboratory experiments
- resolves numerous New Keynesian puzzles (including the FG puzzle)
- the effects of MFFG and FG are **different**

Q: By how much does the fiscal announcement improve the FG effects?

This paper

Nested models with **level-k thinking** (exposition *à la* Farhi and Werning [2019]):

- Representative Agent New Keynesian model (**RANK**)
- Tractable Heterogeneous Agent New Keynesian model (**THANK**)
- Heterogeneous Agent New Keynesian model (**HANK**)

Literature

- **Bounded rationality (BR) and unconventional monetary policy:** Angeletos and Lian [2018], Woodford and Xie [2019], Garcia-Schmidt and Woodford [2019], Gabaix [2020], Angeletos and Sastry [2021], Woodford and Xie [2022], Dobrew et al. [2023b], Iovino and Sergeyev [2023]
- **Incomplete markets (IM) and FG:** Werning [2015], McKay et al. [2016], Hagedorn et al. [2019], Bilbiie [2019], Bilbiie [2020], Acharya and Dogra [2020]
- **IM, BR and monetary policy:** Farhi and Werning [2019], Auclert et al. [2020], Grimaud [2021], Pfäuti and Seyrich [2022], Arias et al. [2023], Dobrew et al. [2023a]
- **IM/BR and fiscal policy:** Farhi et al. [2020], Bianchi-Vimercati et al. [2021], Wolf [2021], Angeletos et al. [2023]
- **Fiscal policy and liquidity trap:** Woodford [2011], Eggertsson [2011], Christiano et al. [2011], Correia et al. [2013]
- **FG puzzle and solutions other that IM and BR:** Campbell et al. [2019], Michaillat and Saez [2021], Del Negro et al. [2023]
- **Public debt and liquidity traps:** Rachel and Summers [2019], Gali [2020], Billi and Walsh [2022], Bhattarai et al. [2023]
- **Household responses to policy announcements:** Agarwal and Qian [2014], Coibion et al. [2023]

RANK: overview of the model

- Environment: Farhi and Werning [2019] + public debt
- Demand block: identical households
- Supply block: perfectly rigid prices (Angeletos and Lian [2018], Farhi and Werning [2019], Bilbiie [2019])
- Government:
  - monetary authority
  - fiscal authority
- Bounded rationality: level-k thinking

Households and monetary policy

Households

Maximization problem:

$$\max_{\{c_t\}_{t=0}^{+\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \cdot u(c_t)$$

$$\forall_{t \geq 0} \; c_t + b_{t+1} = R_t \cdot b_t - T_t + Tr_t + Y_t$$

Utility function:

$$u(c) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}$$

Central bank

Monetary policy rule:

$$R_t = \begin{cases} R & \text{if } \; t \neq \tau, \\ R + dR & \text{if } \; t = \tau. \end{cases}$$

Fiscal policy, market clearing, consumption function

Fiscal policy

Budget constraint:

$$\forall_{t\geq 0} T_t + B_{t+1} = Tr_t + R_t \cdot B_t$$

Fiscal rules:

$$\forall_{t\geq 0} B_t = \bar{B} > 0,\ T_t = T = (R-1)\cdot \bar{B},\ Tr_t = -(R_t - R)\cdot \bar{B}$$

Market clearing

$$\forall_{t\geq 0} c_t = Y_t,\ b_t = B_t$$

Consumption function in PE (arbitrary expectations)

$$\forall_{t\geq 0}\ c_t = C\left(R_t, Y_t, Tr_t, \{R_{t+m}, Y_{t+m}, Tr_{t+m}\}_{m>0}\right).$$

Equilibrium

Definition

The equilibrium under **MFFG** is $\left\{Y_t^k\right\}_{t\geq 0}$ such that given $\left\{R_{t+m}, Tr_{t+m}\right\}_{m\geq 0}$ for each $k \geq 1$:

$$\forall_{t\geq 0}\ Y_t^k = C\left(R_t, Y_t^k, Tr_t, \left\{R_{t+m}, Y_{t+m}^{k-1}, Tr_{t+m}\right\}_{m>0}\right),$$

where $\left\{Y_t^0\right\}_{t\geq 0} = \left\{Y\right\}_{t\geq 0}$ (i.e. level-0 expectations correspond to the steady state equilibrium) such that market clearing conditions, monetary and fiscal rules hold.

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where $\left\{Y_t^0\right\}_{t\geq 0} = \{Y\}_{t\geq 0}$ (i.e. level-0 expectations correspond to the steady state equilibrium) such that market clearing conditions, monetary and fiscal rules hold.

Note: Under **FG**, $\{Tr_{t+m}\}_{m>0}$ is replaced with $\left\{Tr_{t+m}^{k-1}\right\}_{m>0}$ and Tr_t is replaced with Tr_t^k in the equation above and, moreover, I set $\left\{Tr_{t+m}^0\right\}_{m>0} = \{0\}_{m>0}$.

Additional notation 1: measures of policy effectiveness

- The elasticity of output in period 0 (horizon τ, level-k thinking, MFFG):

$$\epsilon(\tau, k) \equiv -\frac{R}{Y} \cdot \frac{dY(\tau, k)}{dR}$$

- Under FG it is denoted by: $\hat{\epsilon}(\tau, k)$
- The impact of the fiscal announcement:

$$\Delta\epsilon(\tau, k) \equiv \epsilon(\tau, k) - \hat{\epsilon}(\tau, k)$$

Additional notation 2: auxiliary objects

- MPC:

$$MPC \equiv \frac{dc_0}{dY_0} = 1 - \beta$$

- iMPC with respect to a one-period-ahead income shock (see Auclert et al. [2023b]):

$$iMPC \equiv \frac{dc_0}{dY_1} = \beta \cdot (1 - \beta)$$

- CDF of the binominal distribution: $F\left(k - 1 | \tau - 1, 1 - \beta\right)$
- PMF of the binominal distribution: $f\left(k - 1 | \tau - 1, 1 - \beta\right)$
- Formulas for F and f: ▸ more
- Figures plotting F and f: ▸ more

Main result in RANK

Theorem

Consider a monetary policy shock dR in period $\tau > 0$ in the RANK model under level-k thinking of order k and under the MFFG. We have:

$$\epsilon(\tau, k) = F(k-1|\tau-1, 1-\beta) \cdot \sigma$$

and the impact of fiscal announcement is:

$$\Delta\epsilon(\tau, k) = f(k-1|\tau-1, 1-\beta) \cdot \frac{iMPC}{1-MPC} \cdot \frac{R \cdot \bar{B}}{C}$$

Main result in THANK

details

Theorem

Consider a monetary policy shock dR in period $\tau > 0$ in the THANK model under level-k thinking of order k and under the MFFG. We have:

$$\epsilon(\tau, k|\delta) = \underbrace{F(k-1|\tau-1, 1-\mathcal{M}) \cdot \sigma}_{\text{intertemporal substitution}}$$

$$\underbrace{- F(k-1|\tau-1, 1-\mathcal{M}) \cdot \frac{iMPC^S(1-s)}{1-MPC^S} \cdot \frac{R \cdot \bar{B}}{C^S}}_{\text{interest earnings}}$$

$$\underbrace{+ F(k-1|\tau-1, 1-\mathcal{M}) \cdot \frac{iMPC^S(\delta)}{1-MPC^S} \cdot \frac{R \cdot \bar{B}}{C^S}}_{\text{transfers (restribution)}}$$

$$\underbrace{+ F(k-2|\tau-1, 1-\mathcal{M}) \cdot (1-\mathcal{M}) \cdot \epsilon(0|\delta)}_{\text{transfers (effects on output)}}$$

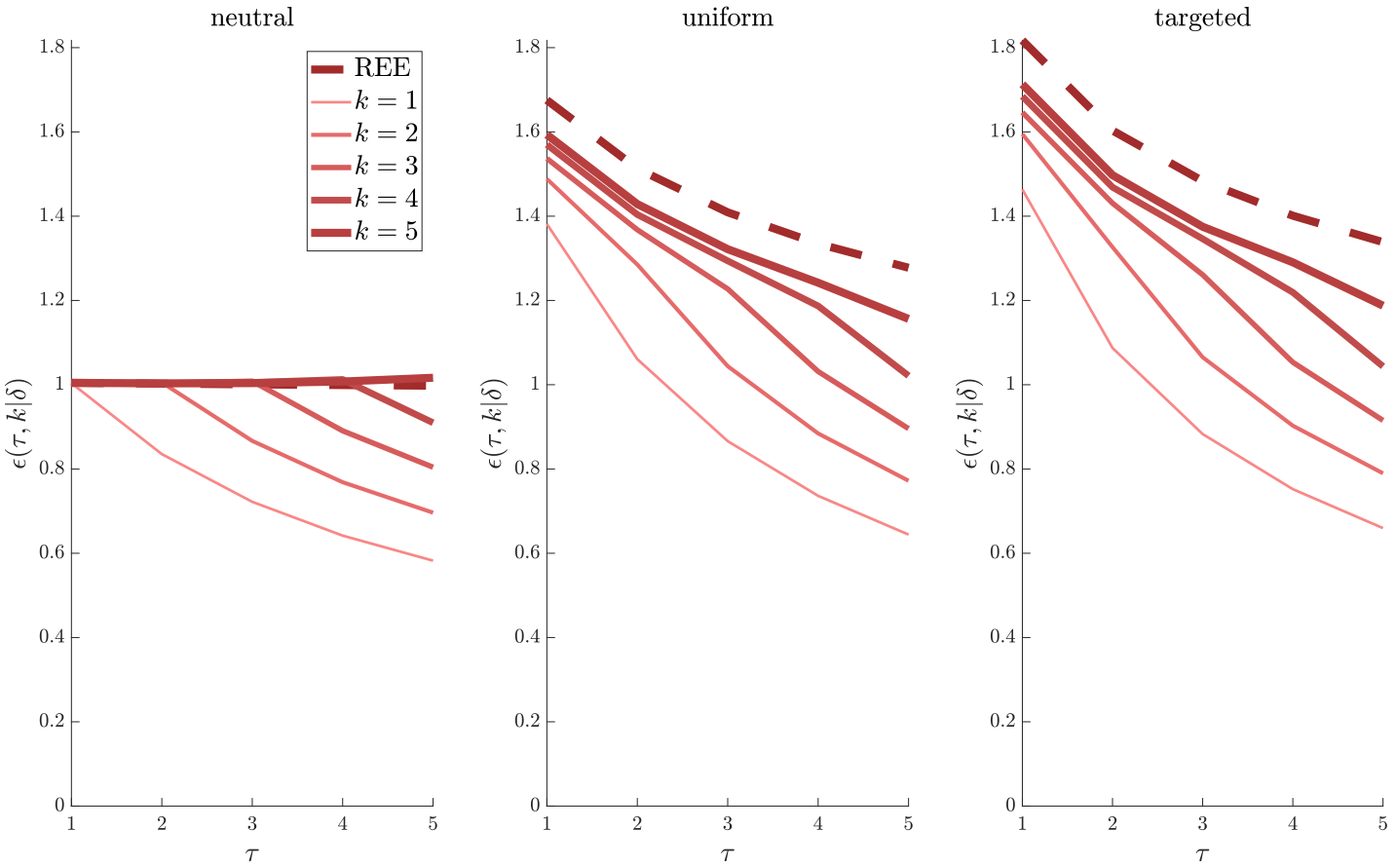
and the impact of fiscal announcement is:

$$\Delta\epsilon(\tau, k|\delta) = f(k-1|\tau-1, 1-\mathcal{M}) \cdot \frac{iMPC^S(\delta)}{1-MPC^S} \cdot \frac{R \cdot \bar{B}}{C^S}$$

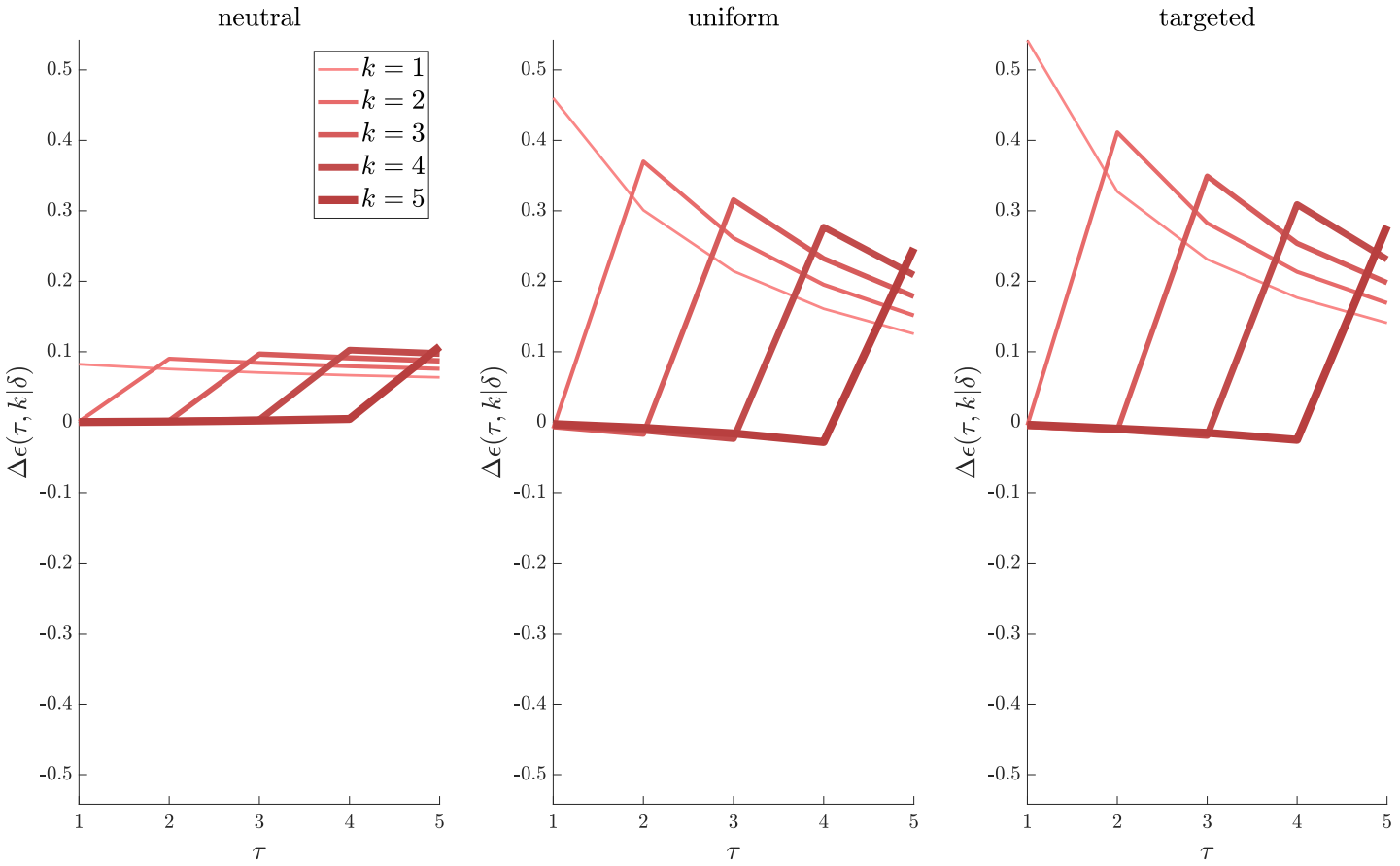
HANK: overview of the model

- Environment: Auclert et al. [2023a] ('canonical' HANK) + level-k thinking
- Demand block: heterogeneous households (wealth and income heterogeneity)
- Supply block: NKPC driven by nominal wage rigidities
- Government:
  - monetary authority
  - fiscal authority
- Bounded rationality: level-k thinking
- HANK = THANK + endogenous wealth distribution
- Details: more
- Calibration: more
- Simulated scenarios: more

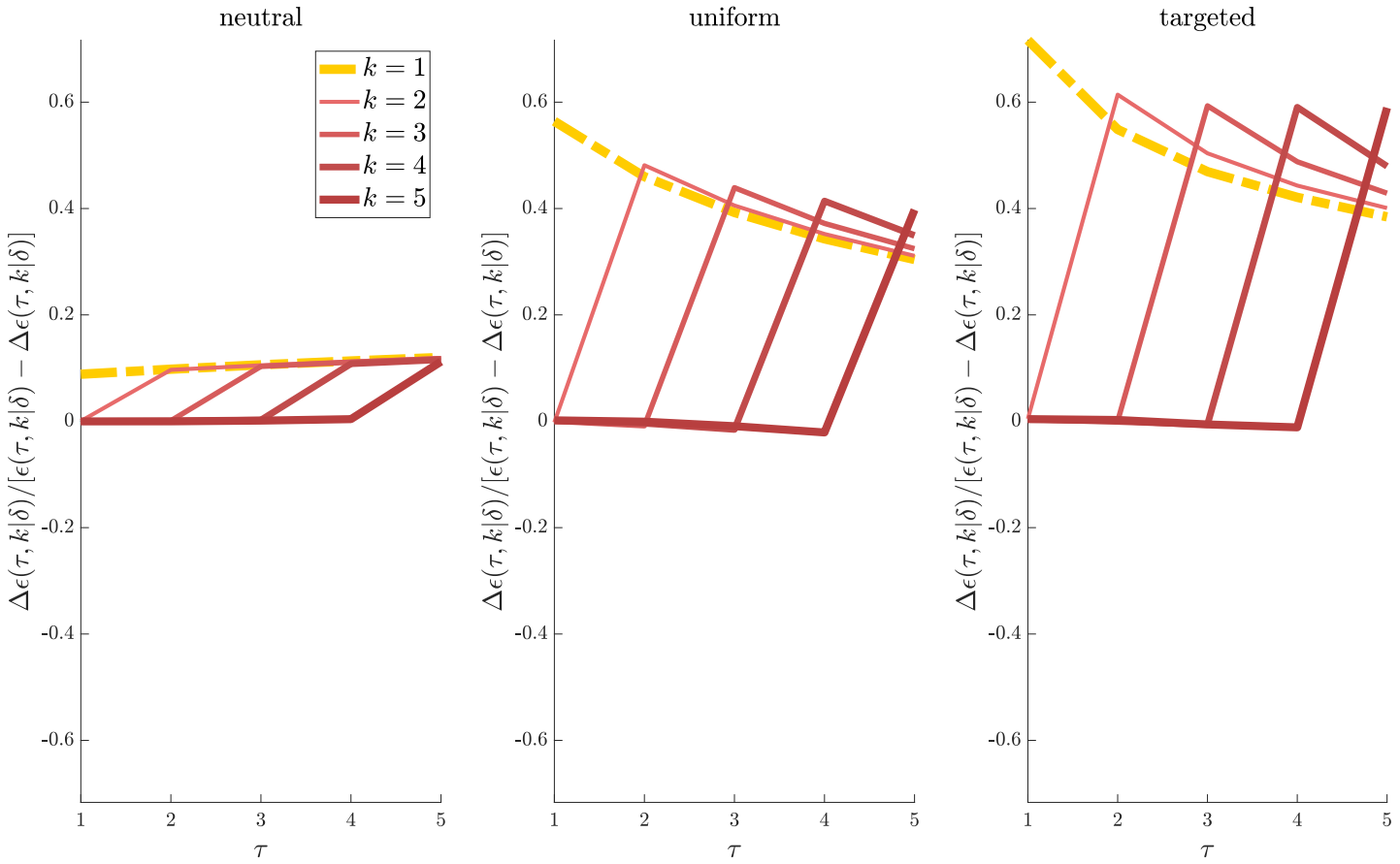
HANK with perfectly rigid wages/prices: interest rate elasticity of output



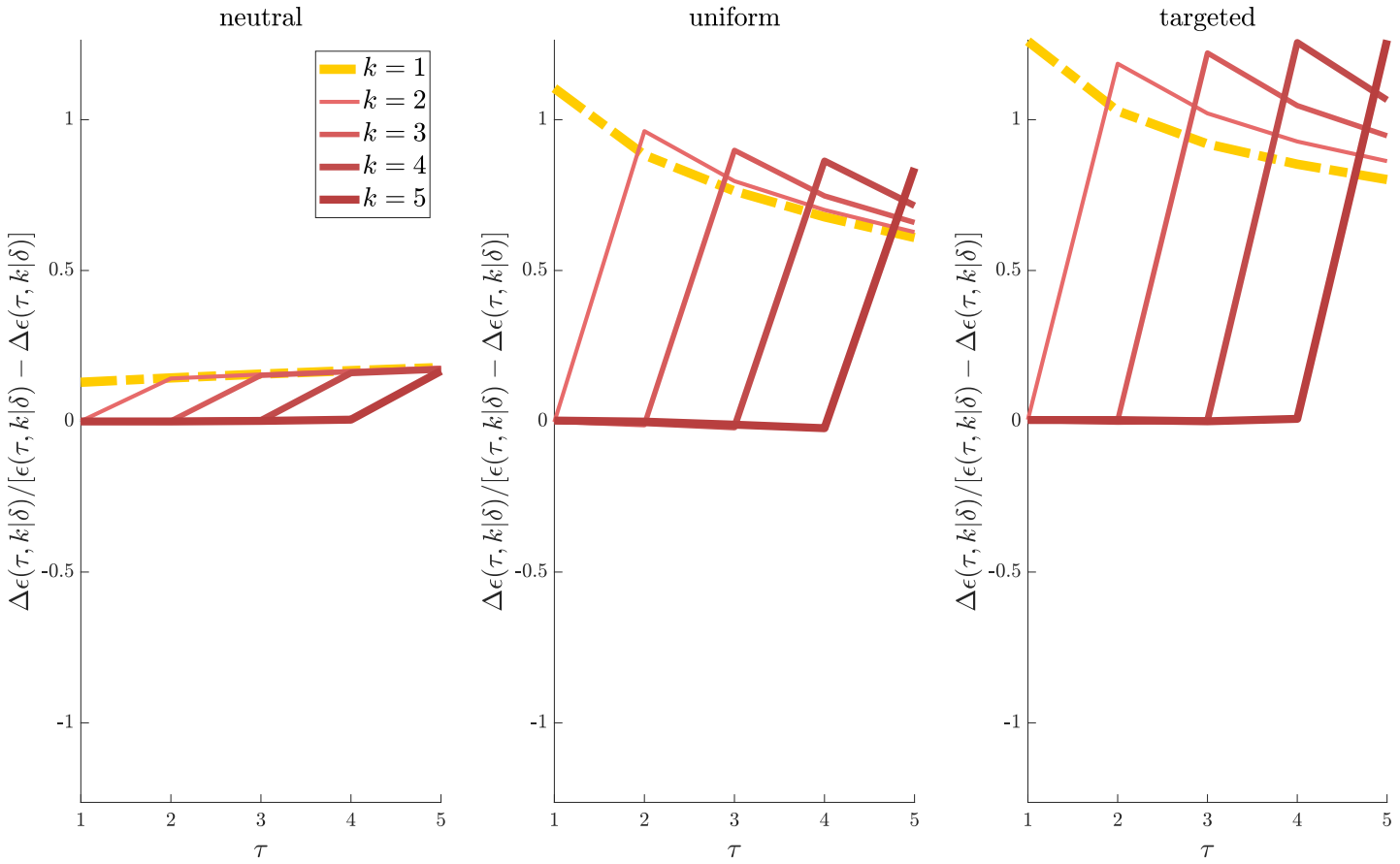
HANK with perfectly rigid wages/prices: contribution of the fiscal announcement to MFFG



HANK with the standard NKPC: relative improvement of the FG effects by the fiscal announcement



HANK with the standard NKPC (high debt): relative improvement of the FG effects by the fiscal announcement



Conclusions

- A theory of coordinated monetary-fiscal announcements
- Application: macro stabilization under constrained monetary and fiscal policies
- Closed-form, intuitive formulas in RANK and THANK
- Improvement of the FG effects (4 quarters) by the fiscal announcement in HANK for $k = 1$:

	uniform	targeted
normal debt	34.2%	42.1%
high debt	68.7%	85.2%

The end

THANK YOU FOR YOUR ATTENTION!

Acknowledgments

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RANK: formulas for f and F

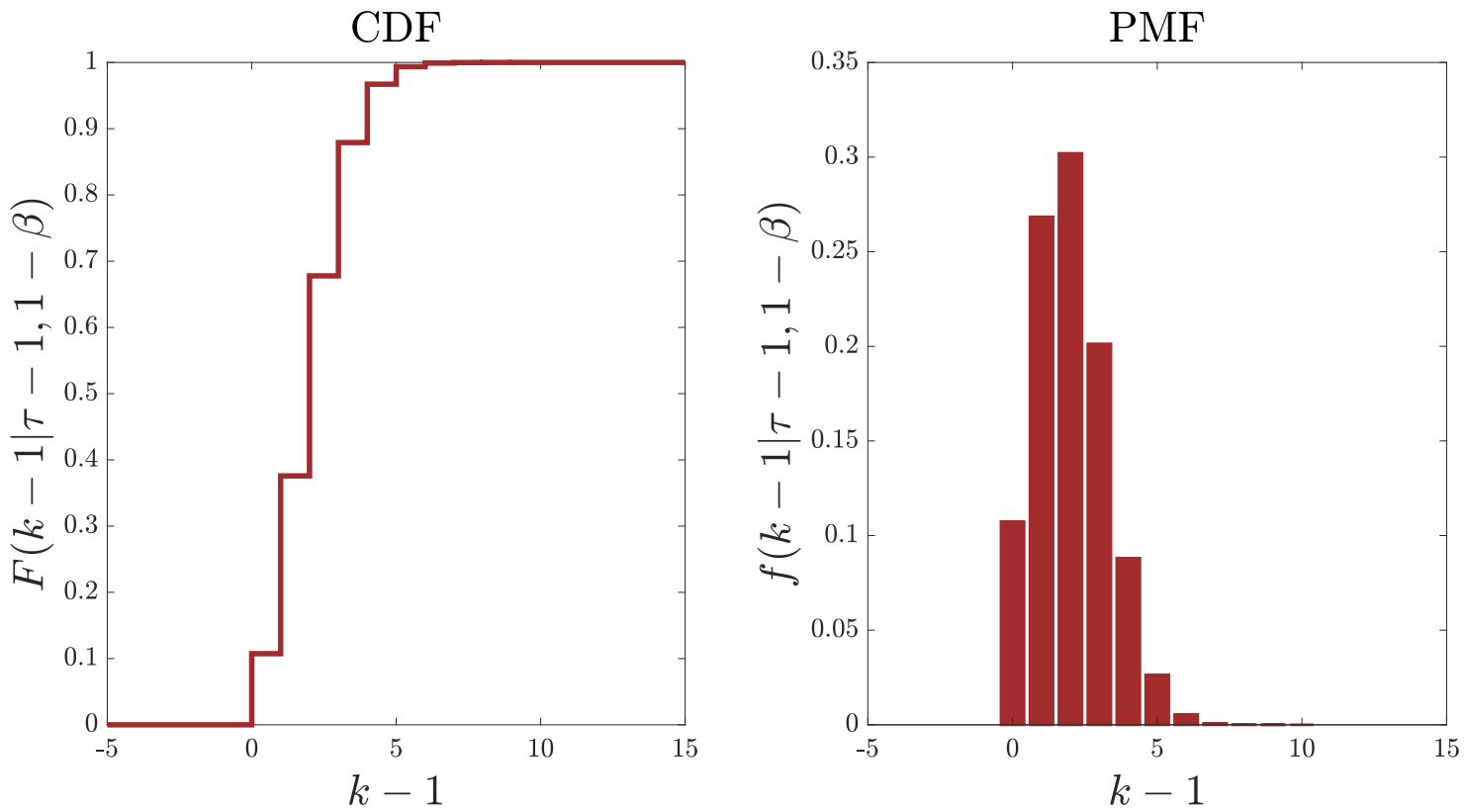
f, F - PMF/CDF of the binominal distribution ($k-1$ successes in $\tau-1$ trials with probability $1-\beta$):

$$f\left(k-1|\tau-1,1-\beta\right)=\left(\begin{array}{c}\tau-1\\k-1\end{array}\right)\cdot\beta^{\tau-k}\cdot(1-\beta)^{k-1}$$

$$F\left(k-1|\tau-1,1-\beta\right)=\sum_{l=0}^{k-1}\left(\begin{array}{c}\tau-1\\l\end{array}\right)\cdot\beta^{\tau-l-1}\cdot(1-\beta)^{l}$$

back

RANK: figures of F and f for $\tau - 1 = 10$ and $\beta = 0.8$



back

Details on the THANK model I

- Probability of staying on island S (H) is s (h)
- Ergodic populations on islands H and S:

$$\lambda = \frac{1-s}{2-s-h},\ 1-\lambda = \frac{1-h}{2-s-h}$$

- Incomes of agents satisfy (see Werning [2015]):

$$\forall_t \ \frac{Y_t^H}{Y_t^S} = \omega \in (0,1)$$

- the beginning-of-period-$t+1$ per capital real asset value on island H (S) is B_{t+1}^H (B_{t+1}^S). We have the following laws of motion:

$$\begin{cases} B_{t+1}^S = s \cdot Z_{t+1}^S + (1-s) \cdot Z_{t+1}^H \\ B_{t+1}^H = (1-h) \cdot Z_{t+1}^S + h \cdot Z_{t+1}^H \end{cases}$$

- taxes levied on household H (S): T_t^H (T_t^S), transfer for household H (S): Tr_t^H (Tr_t^S)

Details on the THANK model II

- Bellman equation:

$$V_t\left(B_t^S, B_t^H\right) = \max_{\left\{c_t^H, c_t^S, Z_{t+1}^H, Z_{t+1}^S\right\}} \left\{(1-\lambda)\cdot u\left(c_t^S\right) + \lambda \cdot u\left(c_t^H\right) + \beta \cdot V_{t+1}\left(B_{t+1}^S, B_{t+1}^H\right)\right\}$$

$$\begin{cases} c_t^S + Z_{t+1}^S - (1-\theta)\cdot B_t^S = R_t \cdot \theta \cdot B_t^S \overbrace{-T_t^S + Tr_t^S + Y_t^S}^{\equiv \Upsilon_t^S} \\ c_t^H + Z_{t+1}^H - (1-\theta)\cdot B_t^H = R_t \cdot \theta \cdot B_t^H \overbrace{-T_t^H + Tr_t^H + Y_t^H}^{\equiv \Upsilon_t^H} \\ Z_{t+1}^S,\ Z_{t+1}^H \geq 0 \end{cases}$$

- As in Bilbiie [2019], I consider the equilibrium with $\forall_t\ Z_t^H = 0$, i.e., when households H are constrained and hand-to-mouth
- A generalization of fiscal policy analyzed in RANK:

$$\forall_{t\geq 0}\ (1-\lambda)\cdot T_t^S + \lambda \cdot T_t^H + B_{t+1} - (1-\theta)\cdot B_t$$

$$= (1-\lambda)\cdot Tr_t^S + \lambda \cdot Tr_t^H + R_t \cdot \theta \cdot B_t$$

- Fiscal rules:

$$\forall_{t\geq 0} B_t = \bar{B} > 0$$

Details on the THANK model III

- Taxes finance steady-state debt service costs (incomplete markets irrelevance holds - see Werning [2015]):

$$T_t^S = \frac{\bar{B}}{1-\lambda} \cdot \left(\bar{R} \cdot s - 1\right), \; T_t^H = \frac{\bar{B} \cdot \bar{R}}{1-\lambda} \cdot (1-h)$$

- Transfers governed by $\delta \in [0,1]$:

$$Tr_t^S = -\frac{1-\delta}{1-\lambda} \cdot (R_t - R) \cdot \bar{B}, \; Tr_t^H = -\frac{\delta}{\lambda} \cdot (R_t - R) \cdot \bar{B}$$

- I consider:

$$\delta = \begin{cases} 1-s & \text{neutral transfers} \\ \lambda & \text{uniform transfers} \\ 1 & \text{targeted transfers} \end{cases}$$

- Market clearing:

$$\forall_{t \geq 0} \lambda \cdot c_t^H + (1-\lambda) \cdot c_t^S = Y_t, \; (1-\lambda) \cdot Z_t^S = B_t$$

Details on the THANK model IV

- Consumption functions:

$$\forall_{t\geq 0}\, c_t^S = C^S\left(R_t, Y_t^S, Tr_t^S, \left\{R_{t+s}, Y_{t+s}^H, Y_{t+s}^S, Tr_{t+s}^H, Tr_{t+s}^S\right\}_{s>0}\right)$$

$$\forall_{t\geq 0}\, c_t^H = \bar{R}\cdot(1-h)\cdot\bar{B} - T_t^H + Tr_t^H + Y_t^H$$

- $\mathcal{M}$ is the largest root of $\Psi\left(\frac{1}{\mathcal{M}}\right) = 0$ where Ψ is a quadratic polynominal with coefficients being functions of the models' parameters (see the paper)
- GE effects of transfers:

$$\epsilon(0|\delta) = -\frac{\bar{B}\cdot\bar{R}\cdot\theta}{Y}\cdot\frac{1-\lambda+\lambda\cdot\omega}{1-\omega}\cdot(1-\delta-s)$$

- MPC of household S:

$$MPC^S \equiv \frac{dc_0^S}{dY_0^S} = 1 - \frac{\mathcal{M}}{\beta\cdot R\cdot s}$$

- iMPC of the S household:

$$iMPC^S(\delta) \equiv \frac{\mathcal{M}}{\beta\cdot\bar{R}\cdot s}\cdot\left[\frac{\beta\cdot\bar{R}\cdot(1-s)}{\omega^{\frac{1}{\sigma}+1}}\cdot\frac{\delta}{\lambda} + \frac{\beta\cdot R\cdot s-\mathcal{M}}{\mathcal{M}}\cdot\frac{1-\delta}{1-\lambda}\right]$$

Details on the THANK model V

- The elasticity of output in period 0 with respect to a monetary policy shock in period τ under level-k thinking of order k under the MFFG and transfers δ:

$$\epsilon\left(\tau, k|\delta\right) = -\frac{\bar{R}}{Y} \cdot \frac{dY\left(\tau, k|\delta\right)}{dR_t}$$

where $dY\left(\tau, k|\delta\right)$ is the reaction of output in period 0 to a shock in period τ. The difference between elasticities under MFFG and FG is denoted by $\Delta\epsilon\left(\tau, k|\delta\right)$.

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Details on the HANK model I

- households face idiosyncratic changes to income productivity y_t governed by a Markovian process featuring a transition probability $\mathbb{P}(y_{t+1}|y_t)$
- Bellman equation:

$$V_t(b,y) = \max_{\{c_t, b_{t+1}\}} \{u(c_t) - v(n_t) + \beta \cdot \mathbb{E}_t V_{t+1}(b_{t+1}, y_{t+1})\}$$

$$\begin{cases} c_t + b_{t+1} - (1-\theta)\cdot b = \frac{R_t \cdot \theta}{\Pi_t} \cdot b - T_t + Tr_t(y|\delta) + y \cdot \frac{W_t}{P_t} \cdot n_t \\ b_{t+1} \geq 0 \end{cases}$$

- where: W_t - nominal wage, P_t - price of consumption goods, $\Pi_t \equiv \frac{P_t}{P_{t-1}}$, y - labor productivity, n_t - hours worked, $Tr_t(y|\delta)$ transfer received by household with productivity y under transfer policy δ
- solution: policy functions $c_t(b,y)$, $b_{t+1}(b,y)$
- As we shall see: $\frac{W_t}{P_t} = 1$, n_t is taken as given by households (so the maximization problem is analogous to the one in RANK and THANK)
- Utility function:

$$u(c) - v(n) = \frac{c^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} - \gamma \cdot \frac{n^{1+\frac{1}{\phi}}}{1 + \frac{1}{\phi}}$$

Details on the HANK model II

- wages negotiated by labor unions
- each union offers a different labor variety j to producers of consumption goods
- labor union maximizes welfare of households subject to labor demand (a generalization of Erceg et al. [2000] developed by Auclert et al. [2023b] and Auclert et al. [2023a]):

$$F_t\left(W_{j,t-1}\right) = \max_{W_{j,t},\, N_{j,t}} \left\{ \int \left(u\left(c_t\right) - v\left(n_t\right)\right) d\mu_t\left(b,y\right) \right.$$

$$\left. -\frac{\psi}{2} \cdot \int \left(\frac{W_{j,t}}{W_{j,t-1}} - 1\right)^2 + \beta \cdot \mathbb{E}_t F_{t+1}\left(W_{j,t}\right) \right\}$$

subject to:

$$N_{j,t} = \left(\frac{W_{j,t}}{W_t}\right)^{-\xi} \cdot N_t$$

- where: $\mu_t\left(b,y\right)$ - distribution of households over assets and productivity levels, ψ - parameter of the quadratic utility cost of wage adjustment, $W_t = \left(\int W_{j,t}^{1-\xi} dj\right)^{\frac{1}{1-\xi}}$ - wage index and ξ governs the substitutability between labor varieties, N_t - aggregate labor

Details on the HANK model III

- solution: the NKPC (see Auclert et al. [2023b] for derivation)

$$\left(\Pi_t^W - 1\right) \cdot \Pi_t^W = \frac{\xi}{\psi} \cdot N_t \cdot \left(v'\left(N_t\right) - \frac{\xi - 1}{\xi} \cdot \int y \cdot u'\left(c_t\left(b, y\right)\right) d\mu_t\left(b, y\right) \right)$$

$$+\beta \cdot \left(\Pi_{t+1}^W - 1\right) \cdot \Pi_{t+1}^W$$

  where $\Pi_t^W \equiv \frac{W_t}{W_{t-1}}$

- perfectly competitive (generate zero profits - analogously to RANK and THANK) and set flexible prices, therefore:

$$P_t = W_t$$

- thus: $\Pi_t^W = \Pi_t$
- production technology: $Y_t = N_t$

Details on the HANK model IV

- budget constraint (analogous to RANK and THANK):

$$\forall_{t\geq 0} T_t + B_{t+1} - (1-\theta)\cdot B_t = Tr_t + \frac{R_t}{\Pi_t}\cdot\theta\cdot B_t$$

where B_t is aggregate government debt. Fiscal rules:

$$\forall_{t\geq 0} B_t = \bar{B} > 0,\ T_t = \left(\frac{R}{\Pi} - 1\right)\cdot\theta\cdot\bar{B},\ Tr_t = -\left(\frac{R_t}{\Pi_t} - \frac{R}{\Pi}\right)\cdot\theta\cdot\bar{B}$$

i.e., as in RANK and THANK: debt is constant, taxes finance steady-state debt service costs, aggregate transfers Tr_t are financed with a windfall resulting from the monetary shock

- redistribution (where $\mathcal{Y}$ is the set of income levels of households to which transfers are sent):

$$Tr_t\left(y|\delta\right) = \begin{cases} \frac{b}{\bar{B}}\cdot Tr_t & \text{neutral transfers} \\ Tr_t & \text{uniform transfers} \\ \frac{1}{\mathbb{P}(y\in\mathcal{Y})}\cdot Tr_t & \text{targeted transfers} \end{cases}$$

Details on the HANK model V

- Monetary rule (see Farhi and Werning [2019]):

$$R_{t+s} = \begin{cases} R & \text{if } s < \tau \\ R - dR & \text{if } s = \tau \\ R \cdot \left(\frac{\Pi_{t+s}}{\Pi}\right)^{\phi_\Pi} & \text{if } s > \tau \end{cases}$$

  where $dR > 0$ is the size of the monetary shock and ϕ_Π is the Taylor rule parameter

- Markovian changes to productivity and optimal saving policies induce the following law of motion:

$$\forall_{t \geq 0}\, \mu_{t+1}(\mathcal{B}, \mathcal{Y}) = \int \left[\mathbb{I}_{\{b_{t+1}(b,y) \in \mathcal{B}\}} \cdot \mathbb{P}\left(y_{t+1} \in \mathcal{Y} | y\right)\right] d\mu_t(b, y)$$

  where $\mathcal{B}$ and $\mathcal{Y}$ are Borel subsets of spaces of assets holdings and labor productivity levels, respectively and $\mathbb{I}$ is the indicator function

- standardization of aggregate labor productivity and the population size:

$$\forall_{t \geq 0} \int y d\mu_t(b, y) = \int d\mu_t(b, y) = 1$$

Details on the HANK model VI

- Labor:
$$\forall_{t\geq 0,j} n_t = N_t = N_{j,t}$$
- Consumption goods:
$$\forall_{t\geq 0} \int c_t(b,y)\, d\mu_t(b,y) = Y_t$$
- Assets:
$$\forall_{t\geq 0} \int b_{t+1}(b,y)\, d\mu_t(b,y) = B_{t+1}$$
- given that $\frac{W_t}{P_t} = 1$, $Y_t = n_t$ (also useful when simplifying the equilibrium definition), and T_t is constant over time, the aggregate consumption can be formulated as:
$$C_t \equiv \int c\left(b, y | R_t, \Pi_t, Tr_t(\cdot|\delta), Y_t, \{R_{t+s}, \Pi_{t+s}, Tr_{t+s}(\cdot|\delta), Y_{t+s}\}_{s>0}\right) d\mu_t(b,y)$$
- Using this formulation, under level-k and the MFFG I define (under the FG Tr_{t+s} is replaced with Tr_{t+s}^{k-1} and Tr_t is replaced with Tr_t^k):
$$C_t^k \equiv \int \underbrace{c\left(b, y | R_t, \Pi_t^k, Tr_t(\cdot|\delta), Y_t^k, \left\{R_{t+s}, \Pi_{t+s}^{k-1}, Tr_{t+s}(\cdot|\delta), Y_{t+s}^{k-1}\right\}_{s>0}\right)}_{\equiv c_t^k(b,y)} d\mu_t^k(b,y)$$

Details on the HANK model VII

- saving policy under level-k is defined as:

$$b_{t+1}^{k}\left(b,y\right)\equiv\left(1-\theta\right)\cdot b+\frac{R_t\cdot\theta}{\Pi_t}\cdot b_t-T_t+Tr_t\left(y|\delta\right)+y\cdot\frac{W_t}{P_t}\cdot n_t-c_t^{k}\left(b,y\right)$$

- define:

$$\Omega_t\equiv\frac{\xi}{\psi}\cdot N_t\cdot\left(v'\left(N_t\right)-\frac{\xi-1}{\xi}\cdot\int y\cdot u'\left(c_t\left(b,y\right)\right)d\mu_t\left(b,y\right)\right)$$

- using $\Pi_t^W=\Pi_t$ rewrite the NKPC as:

$$\left(\Pi_t-1\right)\cdot\Pi_t=\Omega_t+\sum_{s=1}^{+\infty}\beta^s\cdot\Omega_{t+s}$$

- thus, under level-k thinking we have the following condition summarizing the optimal price-setting behavior of labor unions:

$$\left(\Pi_t^k-1\right)\cdot\Pi_t^k=\Omega_t^k+\sum_{s=1}^{+\infty}\beta^s\cdot\Omega_{t+s}^{k-1}$$

Details on the HANK model VIII

Definition

The equilibrium under the MFFG is: $\left\{\Pi_t^k, Y_t^k, \Omega_t^k, \mu_t^k, c_t^k(b,y), b_{t+1}^k(b,y)\right\}_{t\geq 0}$ such that given $\{R_t, Tr_t(y|\delta)\}_{t\geq 0}$ and given $\mu_0^k = \mu_0$ for each $k \geq 1$: given $\{R_{t+s}, Tr_{t+s}(y|\delta)\}_{s\geq 0}$, $\left\{\Pi_{t+s}^{k-1}, Y_{t+s}^{k-1}\right\}_{s>0}$, and $\left\{\Pi_t^k, Y_t^k\right\}$ functions $c_t^k(b,y)$, $b_{t+1}^k(b,y)$ solve household problem for each $t \geq 0$, given $\left\{\Omega_{t+s}^{k-1}\right\}_{s>0}$ and Ω_t^k inflation Π_t^k solves:

$$\left(\Pi_t^k - 1\right) \cdot \Pi_t^k = \Omega_t^k + \sum_{s=1}^{+\infty} \beta^s \cdot \Omega_{t+s}^{k-1},$$

the government budget constraint holds and the monetary policy rule is satisfied, the law of motion of measure μ_t^k is induced by the Markovian process $\mathbb{P}(y_{t+1}|y_t)$ and policy function $b_{t+1}^k(b,y)$, market clearing conditions are satisfied.

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Calibration 1

- Idiosyncratic labor productivity process as in Krueger et al. [2016]:

$$\begin{cases} \log y_{t+1} = \log \hat{y}_t + \epsilon_{y,t+1} \\ \log \hat{y}_{t+1} = \rho \cdot \log \hat{y}_t + \epsilon_{\hat{y},t+1} \end{cases}$$

  where $\epsilon_y \sim N\left(0, \sigma_y^2\right)$, $\epsilon_{\hat{y}} \sim N\left(0, \sigma_{\hat{y}}^2\right)$, $\rho \in (0,1)$

- To match the average MPC, β is assumed to be uniformly distributed:

$$\beta \sim U\left[\underline{\beta}, \overline{\beta}\right]$$

- Targeted transfers: $\mathcal{Y}$ - set of productivities corresponding to bottom 25% income earners

Calibration 2

Parameter	Description	Value	Target/Source
θ	debt maturity	1	one-period debt
ψ	wage-adjustment cost	700	NKPC slope (Auclert et al. [2023a])
σ	intertemporal substitution	1	McKay and Reis [2016]
ξ	substitution between labor varieties	7	Auclert et al. [2023a]
γ	labor disutility parameter	0.86	$Y = N = 1$
ϕ	Frisch elasticity	0.5	McKay and Reis [2016]

Calibration 3

Parameter	Description	Value	Target/Source
ρ	autocorrelation (persistent component)	0.99	Krueger et al. [2016]
$\sigma_{\hat{y}}$	standard error (persistent component)	0.10	Krueger et al. [2016]
σ_y	standard error (transitory component)	0.11	Krueger et al. [2016]
ϕ_Π	Taylor rule parameter	1.5	standard value
$\bar{B}$	government debt	2.2	annual $\bar{B}/(4 \cdot Y)$ (Auclert et al. [2023a])
$\overline{\beta}$	discount factor (patient households)	0.988	annual R/Π of 2%
$\underline{\beta}$	discount factor (impatient households)	0.970	MPC (Auclert et al. [2023a])
R	steady state nominal interest rate	0.005	$\Pi = 1$

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Simulations: HANK

I now report the results for three variants of the HANK model:

- the model with perfectly rigid prices/wages ($\psi \to +\infty$): to bridge the quantitative analysis with the analytical results in the THANK model where the perfect price stickiness was assumed
- the model with the NKPC (benchmark simulation)
- the model with the NKPC with high debt (i.e. $\bar{B}$ doubles when compared to the benchmark) to study the role of debt for the effectiveness of the MFFG

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