The Premia on State-Contingent Debt Instruments[1]

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[1]The views expressed in this paper are our own and do not necessarily represent those of the IMF, its Executive Board or its management.

Introduction

- State-contingent Debt Instrument (SCDI) link a sovereign's debt payments to a country's GDP, to commodity prices, or to natural disasters.

- GDP-linked warrants are most prominent examples:
  1. Argentina (2005)
  2. Greece (2012)
  3. Ukraine (2015)

- Despite the economic rationale for SCDIs (e.g. increased fiscal space), there has been little study to uncover empirical patterns.
  - Chamon et al. (2008) documented a sizable but declining premia.

Overview

- This paper develops a model to analyze and quantify the risk premium of a state-contingent sovereign debt instrument.

- **[Empirics]** Using an extended sample, we reveal three stylized facts:
  1. The premium associated with SCDIs is high and persistent.
  2. The SCDI premium is less counter-cyclical than the default premium in government bond returns.
  3. Liquidity premium in SCDIs is higher and fluctuates more widely.

- **[Theory]** We develop a model of sovereign debt investment with robust preferences a la Hansen and Sargent (2001) to explain the cyclical properties.

Contribution to the Literature

- This paper contributes to the literature primarily by documenting the empirical properties of GDP-linked bonds, extending Chamon et al. (2008).
- The theoretical literature has long predicted the benefits of these instruments while cautioning on the practical obstacles to their issuance.
  - Benefits: Borensztein and Mauro (2004), Hatchondo and Martinez (2012), Bertinatto et al. (2017)
  - Premium Size: Levy (2016), Roch and Roldan (2021)

Outline

1. General Framework
2. Stylized Facts
3. Model
4. Conclusion

General Framework

- We estimate the time-varying premium of a GDP-linked warrant by comparing its value implied by a country's fundamentals with actual price.
- The model-implied price of a GDP-linked warrant is defined as

$$\hat{p}_m(r_m^{scdi}) = \mathbb{E}_m \left[\sum_{t>=m}^{T} \frac{c_t(j)}{1 + r_{t,m} + r_m^{scdi}} \right]$$

  where m indexes month, t indexes year and $c_t(j)$ is annual coupon payment.

- $c_t(j)$ is a function of nominal GDP, real GDP growth, and exchange rate between local and foreign currencies. The formula vary across countries.

Example 1: Argentina (2005)

- Annual coupons of the Argentinian GDP warrants are determined as follows:

$$c_t^{ARG}(j) = \frac{\gamma(j)}{20 e_t(j)}(Y_t - Y_t^c) D_t \times \mathbb{I}_{\{Y_t > Y_t^c\}} \times \mathbb{I}_{\left\{Y_t/Y_{t-1} > Y_t^c/Y_{t-1}^c\right\}}$$

where

Y_t : Nominal GDP in the 1993 peso price

Y_t^c : Cutoff in the contract

D_t : GDP Deflator

$e_t(j)$: Exchange rate between the peso and currency j

$\gamma(j)$: Currency adjustment coefficient

- The formula show that GDP warrants are characterized by two features (i) payment cutoff and (ii) linear indexation to $Y_t - Y_t^c$.

- We run a Monte Carlo simulation to simulate coupon payments. The real GDP growth evolves according to

$$y_{t+1} - \hat{y}_{m,t+1} = \theta_y(y_t - \hat{y}_{m,t}) + \varepsilon_{y,t} \quad \text{for} \quad m \leq t \leq T$$

where $y_t \equiv \log(Y_t/Y_{t-1})$ and $\hat{y}_{m,t}$ is a forecast for year t at time m.

- The log change of the GDP deflator follows

$$d_t = \hat{d}_{m,t} + \varepsilon_{d,t} \quad \text{for} \quad m \leq t \leq T$$

where $\varepsilon_{y,t}$ and $\varepsilon_{d,t}$ are error components drawn from a bi-variate normal distribution $N(0, \Sigma_{y,d})$.

- Finally, nominal exchange rate is assumed to follow

$$e_t = \hat{e}_{m,t} + \varepsilon_{e,t} \quad \text{for} \quad m \leq t \leq T$$

where $\varepsilon_{e,t}$ is drawn from $N(0, \sigma_e^2)$.

Calibration

- **GDP Growth Forecasts**: market consensus of GDP growth rates published by Consensus Economics.
- **GDP Deflator Forecasts**: IMF World Economic Outlook database. Extrapolated from year 5 onwards
- **Exchange Rate Forecasts**: 1-year and 2-year forward exchange rates. Extrapolated from year 3 onwards (constant)

Table 1: Parameter Values

	Paremter Values			
Country	θ_y	$\Sigma_{y,d}$	Period	Milestone
Argentina	0.7239	[0.003, -0.001; -0.001, 0.006]	1992-2005	Hyperinflation ended in 1992
Greece	0.2296	[0.001, 0.000; 0.000, 0.000]	2002-2018	Joined Euro Zone in 2001
Ukraine	0.6553	[0.004, -0.003; -0.003, 0.01]	1995-2016	Hyperinflation ended in 1995

Discount Rates

- The discount rate consists of three components.
  1. **Standard discount rate**: Risk-free interest rate plus CDS spread
  2. **Residual discount rate**: $r_m^{scdi}(j)$, which equates the market price of a security with the model-implied price estimate such that

$$p_m(j) = \hat{p}_m^{bid}(j, r_m^{scdi}(j)) \tag{1}$$

     where $p_m^{bid}(j)$ indicates the bid price and $\hat{p}_m(j, r_m^{scdi}(j))$ the model-implied price.

  3. **Liquidity premium**: $r_m^{liq}(j)$, is defined as the spread equating

$$p_m^{ask}(j) = \hat{p}_m(j, r_m^{scdi}(j) + r_m^{liq}(j)) \tag{2}$$

- We interpret $\{r_m^{scdi}(j)\}_m$ as the additional time-varying premium for buyers who intend to hold the GDP-linked warrant until maturity.

Algorithm

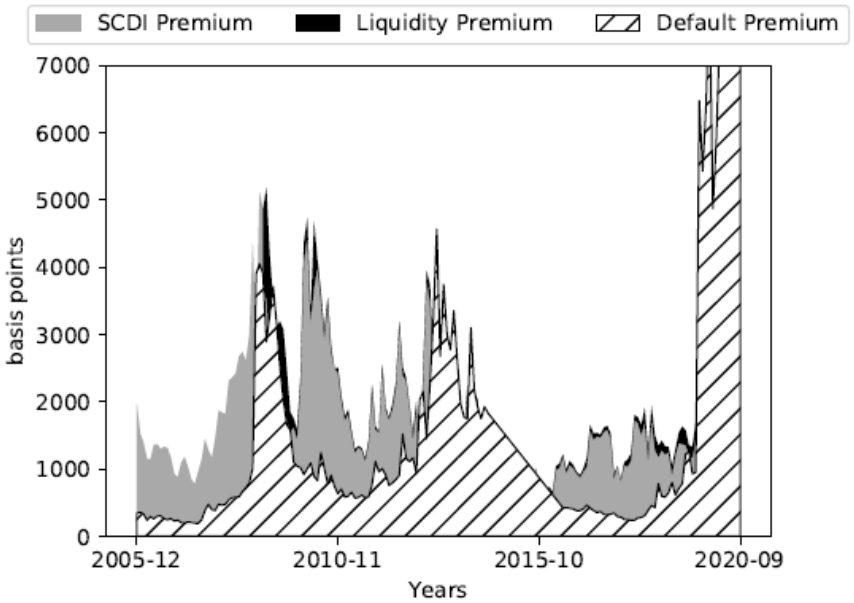
- Our final step is to run Monte-Carlo simulations to measure the discounted value of a GDP-linked warrant and derive its premia.
  1. Calibrate θ, Σ, and σ_e prior to running simulations.
  2. For each month m in year T, extract market forecast data $\{\hat{y}_{m,T+t}\}_{t=0,1\ldots,30}$, $\{\hat{d}_{m,T+t}\}_{t=0,1\ldots,30}$ and $\{\hat{e}_{m,T+t}\}_{t=0,1,2}$. Draw random numbers and compute g_t, d_t and e_t until the maturity year.
  3. For each month m, compute discounted expected values of the GDP-linked warrant.
  4. Find r^{scdi} and r^{liq} such that the discounted values are equated to the actual trading prices.

Stylized Facts

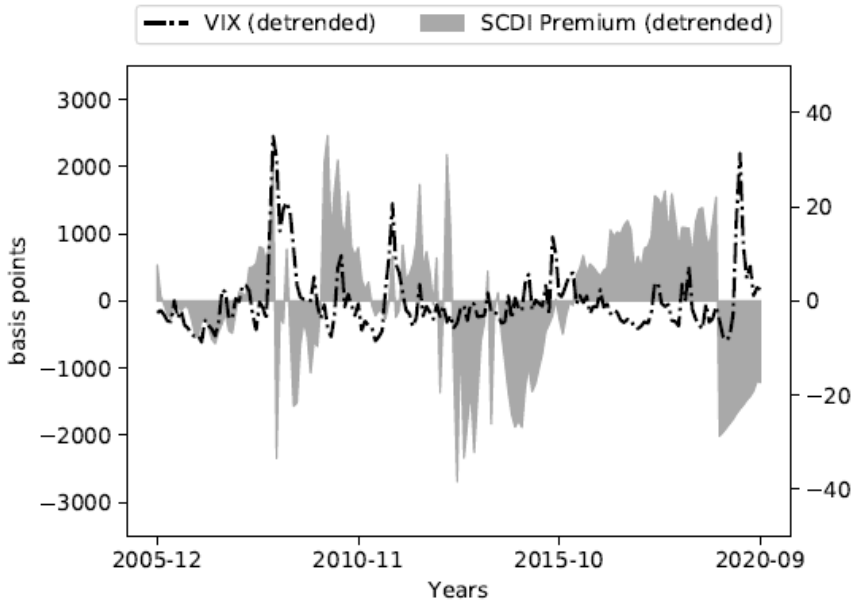
- **Stylized Fact 1.** SCDI Premium is high, persistent and shows no sign of a downward trend over the first five years since issuance.

Figure 1: Argentina

(c) Premium Decomposition



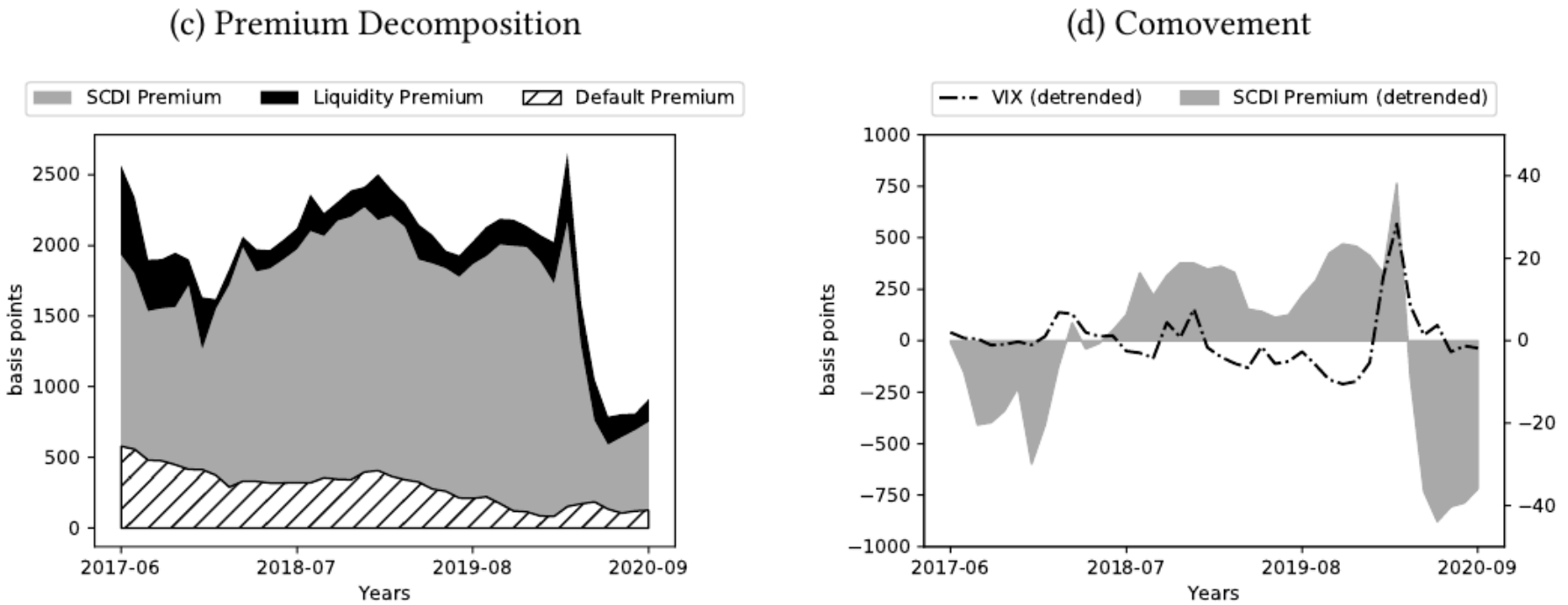
(d) Comovement



Stylized Facts

- **Stylized Fact 1.** SCDI Premium is high, persistent and shows no sign of a downward trend over the first five years since issuance.

Figure 2: Greece

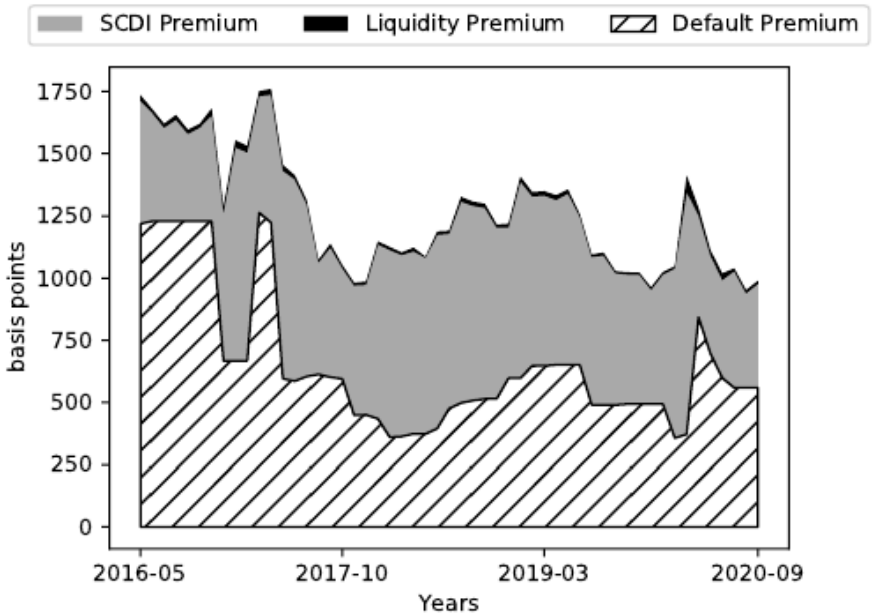


Stylized Facts

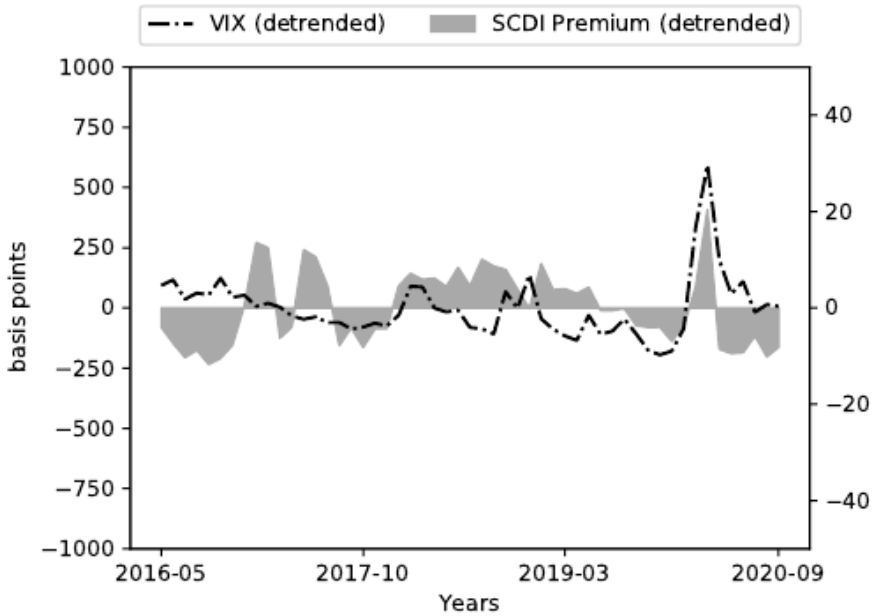
- **Stylized Fact 1.** SCDI Premium is high, persistent and shows no sign of a downward trend over the first five years since issuance.

Figure 3: Ukraine

(c) Premium Decomposition



(d) Comovement

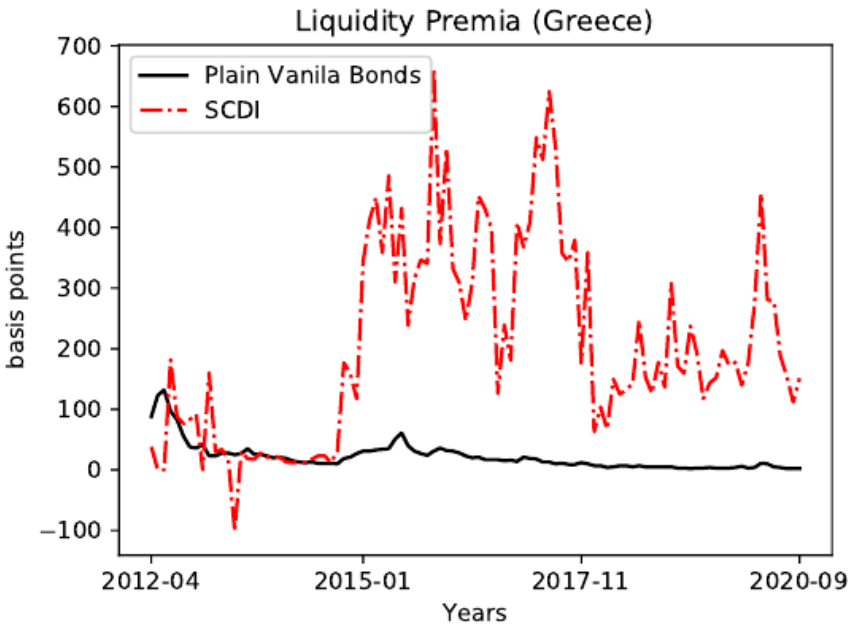


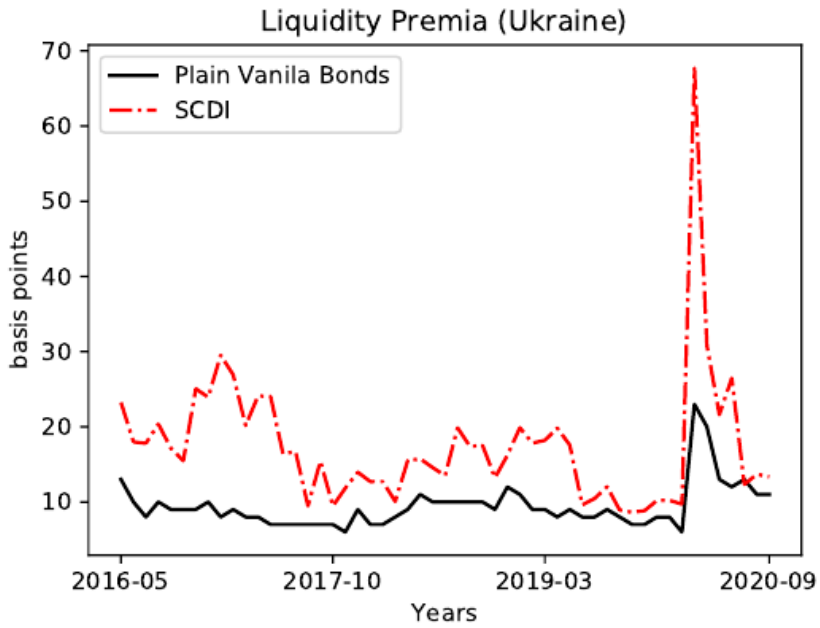
- **Stylized Fact 2.** The SCDI premium is less counter-cyclical than the default premium in government bond returns.

Table 4: Contemporaneous Correlations

Premium Type	Horizon	*(a) Domestic*			*(b) Global*	
		IP	Unemp.	Earning Yields	VIX	US MP Shock
SCDI Premium	12m	0.1420				
		(0.0541)				
	6m	0.0208				
		(0.0546)				
	1m	0.0443	0.0274	-0.1749	0.0115	0.0236
		(0.0546)	(0.0795)	(0.0783)	(0.0796)	(0.0795)
Default Premium	12m	-0.1647				
		(0.0539)				
	6m	-0.1001				
		(0.0544)				
	1m	-0.0069	0.2769	-0.0500	0.0806	0.0267
		(0.0546)	(0.0764)	(0.0795)	(0.0793)	(0.0795)

- **Stylized Fact 3.** Liquidity premium in GDP-linked warrant markets is higher and fluctuates more widely than liquidity premium in plain-vanilla government bond markets.





Model

- We develop a model of sovereign debt investment with robust preferences to account for the cyclical properties of the SCDI premium.
- Investors with robust preferences are concerned that their forecasting model is misspecified, exaggerating the likelihood of a bad state.
- **Intuition**:
  1. A higher premium is required when a small deviation from the investors' forecast can change the coupon size widely in the next period.
  2. The payment volatility of plain-vanilla bond is maximized when the economy turns to recession due to the increased risk of default.
  3. By contrast, the payment volatility of GDP-linked warrants rises during good times when the current GDP is near the payment threshold.

(1) Setup

- There is a mass 1 of international lenders. Each lender has a utility function of the form

$$(1-\beta)U_t = (1-\beta)\log(C_t) + \frac{\beta}{1-\gamma}\log(\mathbb{E}_t[exp\{(1-\beta)(1-\gamma)U_{t+1}\}]$$

- As $\gamma \to 1$, the mental cost of considering model deviations becomes prohibitively high $\Rightarrow$ the rational expectations framework.

- The lenders are hand-to-mouth households. Their consumption process is given by

$$c_{t+1} - \bar{c} = \theta_c(c_t - \bar{c}) + \varepsilon_{c,t} \tag{3}$$

- Next, we model the output process of a small open economy. The output growth evolves as follows:

$$g_{t+1} - \bar{g} = \theta_y(g_t - \bar{g}) + \varepsilon_{y,t} \tag{4}$$

- Plain-vanilla bond gives investors a fixed interest rate perpetually. The government defaults automatically if the growth rate falls below a threshold.

$$r^{PV}(g_t) = \begin{cases} \bar{r} & \text{if } g_t \geq \underline{g}^{PV} \\ 0 & \text{Otherwise} \end{cases}$$

- GDP-linked warrant provides variable coupons. The size is proportional to the gap between realized growth rate and a threshold.

$$r^{SCDI}(g_t) = \begin{cases} \alpha(g_t - \underline{g}^{SCDI}) & \text{if } g_t \geq \underline{g}^{SCDI} \\ 0 & \text{Otherwise} \end{cases}$$

- We assume that $\underline{g}^{SCDI} > \bar{g} > \underline{g}^{PV}$.

- The stochastic discount factors are used to price the two government securities. Let $s_t \equiv (c_t, g_t)$ denote the pair of state variables at period t.

- The market prices of PV and SCDI are determined implicitly by the following equations

$$p^{SCDI}(s_t) = \mathbb{E}_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \left(\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega)]} \right) (r^{SCDI}(s_{t+1}) + p^{SCDI}(s_{t+1})) \right]$$

$$p^{PV}(s_t) = \mathbb{E}_t \left[\left(\beta \frac{C_t}{C_{t+1}} \right) \left(\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega)]} \right) (r^{PV}(s_{t+1}) + p^{PV}(s_{t+1})) \right]$$

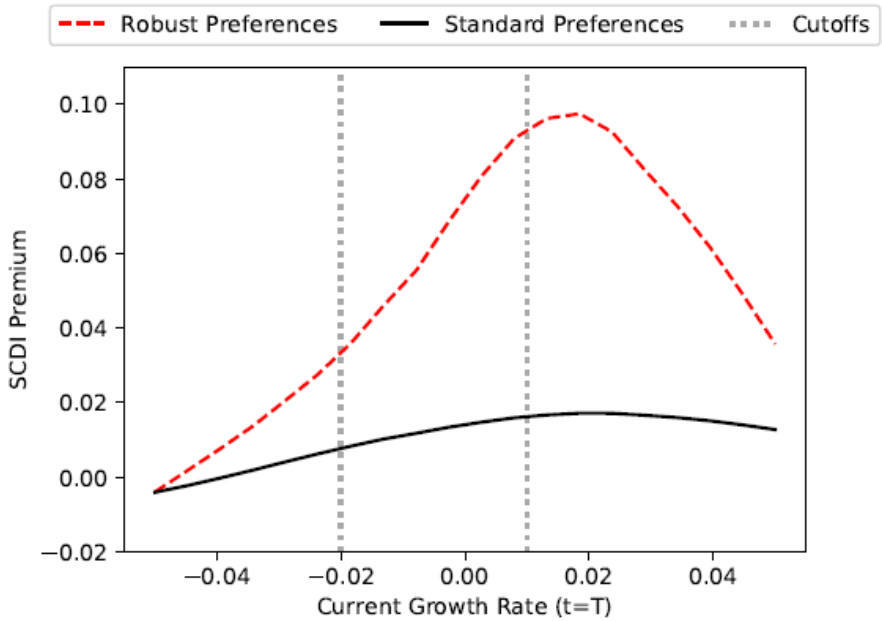
  where $\omega = -\frac{1}{(1-\beta)(1-\gamma)}$.

- Note that $\frac{exp(-U_{t+1}/\omega)}{E_t[exp(-U_{t+1}/\omega]}$ is added to the standard consumption Euler equation because of the robust preferences of investors.

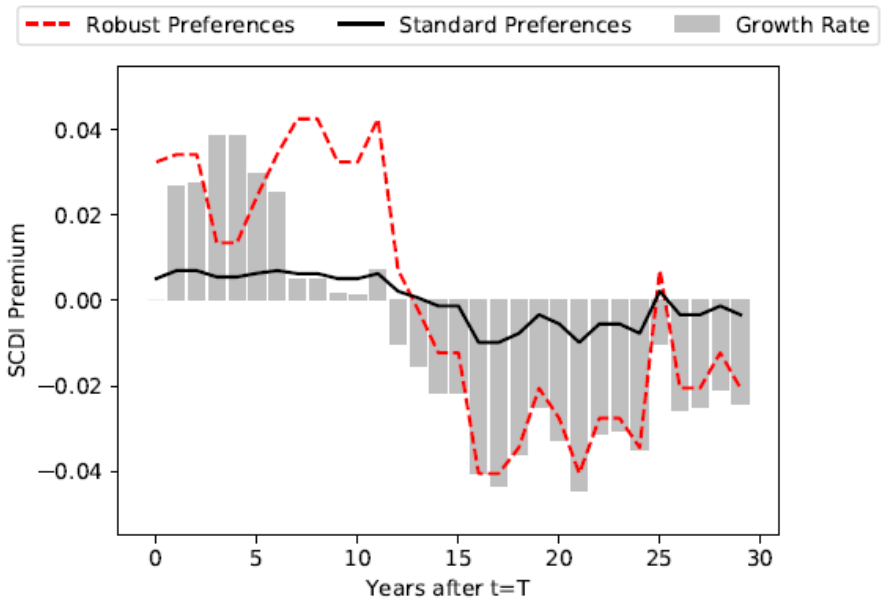
(2) Numerical Example

- The rational expectation model is not sufficient to generate a meaningful size of the SCDI premium.
- Under robust preferences, the premium is highest when g_t is close to $\underline{g}^{SCDI}$ (Panel a) The demeaned premium fluctuates widely along business cycles (Panel b)

(a) SCDI Premia



(b) Price Path



Conclusion

- Extending Chamon et al. (2008), this paper is one of the first attempts to uncover empirical properties of SCDIs.
  1. The risk premium is high. Ambiguity aversion could be one explanation.
  2. The issuance of GDP-linked warrants is most effective when the economy is at the trough of a business cycle.
- We developed a modeling framework to understand the cyclical properties of SCDIs.
- As more countries consider issuing GDP-linked warrants, a more fruitful discussion on cross-country characteristics would become feasible.
- Tentative takeaway: Choosing appropriate payment thresholds, or perhaps avoiding discrete payment thresholds altogether, could lower the SCDI premium. Also, countries may want to market SCDIs to particular classes of investors, i.e. those that are less averse to ambiguity (say, because they hold a well-diversified portfolio of SCDIs issued by heterogeneous countries).