Endogenous Production Networks and Non-Linear Monetary Transmission

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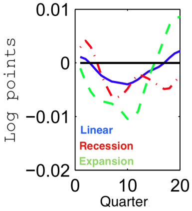
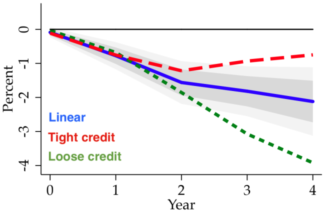
Motivation: non-linear monetary transmission to GDP

Recession vs Expansion



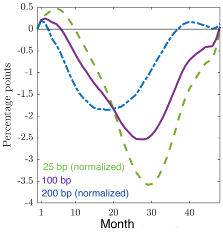
Tenreyro and Thwaites (2016)

Tight vs Loose credit



Jordà et al. (2019)

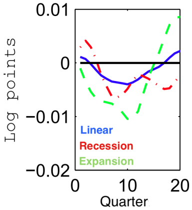
Large vs Small shocks



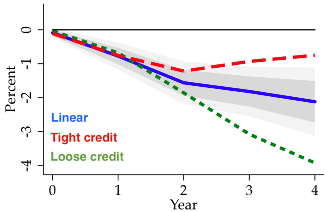
Ascari and Haber (2019)

Motivation: non-linear monetary transmission to GDP

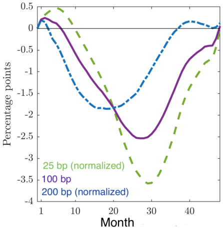
Recession vs Expansion	***Tight vs Loose credit***	***Large vs Small shocks***



Tenreyro and Thwaites (2016)

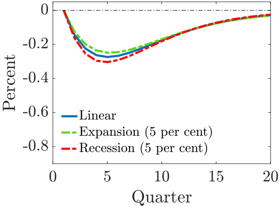


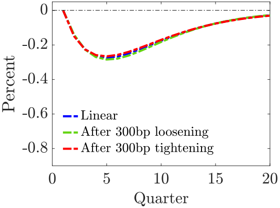
Jordà et al. (2019)

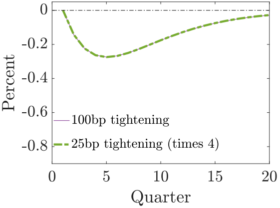


Ascari and Haber (2019)

- 100bp tightening in a fully non-linear medium-scale New Keynesian Model:







This Paper

- A novel tractable framework to rationalize a range of non-linearities in monetary transmission, with the key mechanism supported by evidence using aggregate, sectoral and firm-level data

1. Develop sticky-price New Keynesian model with input-output linkages across sectors that are formed endogenously
    - *Key novel mechanism*: state-dependent strength of complementarities in price setting

2. Jointly rationalize empirically established monetary *non-linearities*:
    - *Cycle dependence*: monetary policy's effect on GDP is *procyclical* (Tenreyro and Thwaites, 2016; Jorda et al., 2019; Alpanda et al., 2019)
    - *Path dependence*: monetary policy's effect on GDP is stronger following *past loose monetary policy* (Jorda et al., 2019)
    - *Size dependence*: large monetary shocks a have disproportionate effect on GDP (Ascari and Haber, 2019)

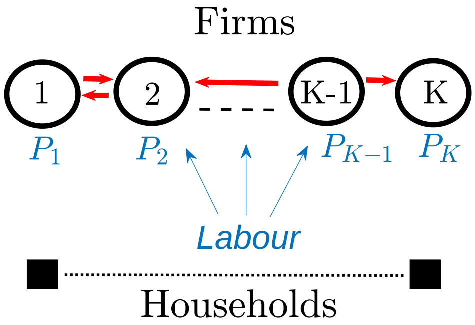
3. Novel model-free empirical evidence on network responses to shocks

Contribution to the literature

- **Endogenous production networks in macroeconomics**: Carvalho and Voightlaender (2015); Oberfield (2018); Taschereau-Dumouchel (2019); Acemoglu and Azar (2020)
  - *Contribution 1*: first model with endogenous production networks and nominal rigidities
  - *Contribution 2*: model-free econometric evidence on network responses to identified productivity and monetary shocks
- **State dependence in monetary transmission**: Tenreyro and Thwaites (2016); Berger et al. (2018); Jorda et al. (2019); Ascari and Haber (2019); Alpanda et al. (2019); Eichenbaum et al. (2019); McKay and Wieland (2019)
  - *Contribution 3*: first framework to use cyclical variation in the shape of the network to jointly rationalize the observed state dependence in monetary transmission

A TWO-PERIOD MODEL

Model primitives



Firms: production and choice of suppliers

- K sectors, continuum of firms Φ_k in each sector
- *Roundabout Production (for firm j in sector k)*:

$$Y_k(j) = \psi(S, \Omega)\mathcal{A}_{k,0}(S_k)N_k(j)^{1-\sum_{r \in S_k} \omega_{kr}} \prod_{r \in S_k} Z_{kr}(j)^{\omega_{kr}}, \quad \forall k, \forall j \in \Phi_k$$

where $S_k \subset \{1, 2, ..., K\}$ is sector k's choice of suppliers, $\mathcal{A}_{k,0}(.)$ is the technology mapping, $\omega_{kr} = [\Omega]_{kr}$ are input-output weights

- *Marginal Cost (conditional on supplier choice)*:

$$MC_k = \frac{1}{\mathcal{A}_{k,0}(S_k)} W^{1-\sum_{r \in S_{kt}} \omega_{kr}} \prod_{r \in S_k} P_r^{\omega_{kr}}, \qquad \forall k, \forall j \in \Phi_k$$

- *Optimal Network*:

$$S_k^* \in \arg\min_{S_k} MC_k(S, P), \qquad \forall k$$

where $S = [S_1, S_2, ..., S_K]'$ and $P = [P_1, P_2, ..., P_K]'$

Firms: pricing under nominal rigidities

- *Profit maximization*:

$$\max_{P_k^*(j)} \Pi_k(j) = [P_k^*(j) Y_k(j) - (1+\tau_k) MC_k Y_k(j)] \quad \text{s.t.}$$

$$Y_k(j) = \left(\frac{P_k(j)}{P_k} \right)^{-\theta} Y_k$$

- *Optimal price*:

$$\overline{P}_k = (1+\mu_k) MC_k, \qquad (1+\mu_k) = (1+\tau_k) \frac{\theta}{\theta - 1}, \qquad \forall k, \forall j \in \Phi_k$$

- *Calvo lotteries (probability of non-adjustment α_k)*:

$$P_k = \left[\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k) \left\{ \frac{1+\mu_k}{\mathcal{A}_{k,0}(S_k)} W \prod_{r \in S_k} \left(\frac{P_r}{W} \right)^{\omega_{kr}} \right\}^{1-\theta} \right]^{\frac{1}{1-\theta}}, \quad \forall k$$

Households and Monetary Policy

- *Flow Utility*: $\mathcal{U} = \log C - N$
- *Consumption Aggregation*: $C \equiv \prod_{k=1}^{K} C_k^{\omega_{ck}}.$
- *Sectoral Consumption Demand*: $C_k = \omega_{ck} \left(\frac{P_k}{P^c}\right)^{-1} C$
- *Cash-in-Advance Constraint*: $P^c C = \mathcal{M} \quad \Rightarrow \quad P_k C_k = \omega_{ck} \mathcal{M}$
- *Money supply rule*: $\mathcal{M} = \mathcal{M}_0 \exp(\varepsilon^m)$

Equilibrium

Definition (Equilibrium)

Equilibrium is a set of prices, allocations and networks such that:

$$P_k^* = \left[\alpha_k P_{k,0}^{1-\theta} + (1-\alpha_k)\left\{\frac{1+\mu_k}{\mathcal{A}_{k,0}(S_k^*)}\mathcal{M}\prod_{r\in S_k^*}\left(\frac{P_r^*}{\mathcal{M}}\right)^{\omega_{kr}}\right\}^{1-\theta}\right]^{\frac{1}{1-\theta}}, \qquad \forall k$$

$$S_k^*(\mathcal{A}_0,\mathcal{M}) \in \arg\min_{S_k}\left[\frac{1}{\mathcal{A}_{k,0}(S_k)}\mathcal{M}\prod_{r\in S_k}\left(\frac{P_r^*}{\mathcal{M}}\right)^{\omega_{kr}}\right], \qquad \forall k$$

$$C_k^* = \omega_{ck}\left(\frac{P_k^*}{\mathcal{M}}\right)^{-1}, \quad C^* \equiv \prod_{k=1}^{K}(C_k^*)^{\omega_{ck}}, \quad \mathcal{M} = \mathcal{M}_0\exp(\varepsilon^m) \qquad \forall k$$

and markets clear, given an initial state $(\mathcal{A}_0, \mathcal{M}_0)$.

Definition (Baseline)

Baseline is the set of prices, allocations and networks consistent with equilibrium under monetary shock at its expected value ($\varepsilon^m = 0$)

BASELINE

Baseline: a two-sector example

- Two sectors: $\omega_{kk} = 0, \quad \tau_k = -\frac{1}{\theta}, \quad \theta \to 1^+, \quad \forall k = 1, 2$

	Sector 1	Sector 2
$a_0(.)$	$a_{1,0}(\varnothing) = 1, \quad a_{1,0}(\{2\}) = \varepsilon^a$	$a_{2,0}(\varnothing) = 1, \quad a_{2,0}(\{1\}) = \varepsilon^a$
Ω	$\omega_{12} = \omega_{c1} = 0.5$	$\omega_{21} = \omega_{c1} = 0.5$
α	$\alpha_1 = 0$	$\alpha_2 = 0.5$

- Real marginal costs: $(mc_{k,0} - m_0) = -a_{k,0}(S_{k,0}) + \mathbf{1}_{-k \in S_{k,0}} \frac{1}{2}(p_{-k,0} - m_0)$

- Optimal network choice over (real) marginal costs $(mc_k - m_0)$:

	$S_2 = \varnothing$	$S_2 = \{1\}$
$S_1 = \varnothing$	$(-1, -1)$	$(-1, -\varepsilon^a - \frac{1}{2})$
$S_1 = \{2\}$	$(-\varepsilon^a - \frac{1}{4} m_0 - \frac{1}{4}, -1)$	$\left(\frac{2}{7}\left\{-5\varepsilon^a - m_0\right\}, \frac{2}{7}\left\{-6\varepsilon^a - 0.5 m_0\right\}\right)$

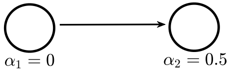
Recession vs Expansion

Recession: $\varepsilon^a = 0$

	$\varnothing$	$\{1\}$
$\varnothing$	$(-1, -1)$	$(-1, -\frac{1}{2})$
$\{2\}$	$(-0.25, -1)$	$(0, 0)$

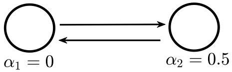
Normal: $\varepsilon^a = 0.65$

	$\varnothing$	$\{1\}$
$\varnothing$	$(-1, -1)$	$(-1, -1.15)$
$\{2\}$	$(-0.9, -1)$	$(-0.92, -1.11)$



Expansion: $\varepsilon^a = 0.8$

	$\varnothing$	$\{1\}$
$\varnothing$	$(-1, -1)$	$(-1, -1.30)$
$\{2\}$	$(-1.05, -1)$	$(-1.14, -1.37)$



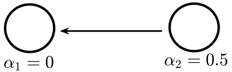
Tight vs Loose money

Tight money: $m_0 = 0$

	$\varnothing$	$\{1\}$
$\varnothing$	$(-1, -1)$	$(-1, -\frac{1}{2})$
$\{2\}$	$(-0.25, -1)$	$(0, 0)$

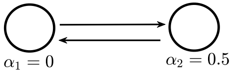
Normal money: $m_0 = 4$

	$\varnothing$	$\{1\}$
$\varnothing$	$(-1, -1)$	$(-1, -\frac{1}{2})$
$\{2\}$	$(-1.25, -1)$	$(-1.14, -0.57)$



Loose money: $m_0 = 8$

	$\varnothing$	$\{1\}$
$\varnothing$	$(-1, -1)$	$(-1, -\frac{1}{2})$
$\{2\}$	$(-2.25, -1)$	$(-2.28, -1.14)$



Baseline: density of the network and activity

Lemma (Expansionary vs Recessionary Baseline)

Suppose the production function is quasi-submodular in $(S_k, \mathcal{A}_{k,0}(S_k)), \forall k$*; then for any two initial technology mappings such that* $\overline{\mathcal{A}_0} \geq \underline{\mathcal{A}_0}$ *it holds that:*

$$S_k^*(\overline{\mathcal{A}_0}, \mathcal{M}_0) \supseteq S_k^*(\underline{\mathcal{A}_0}, \mathcal{M}_0) \qquad C_k^*(\overline{\mathcal{A}_0}, \mathcal{M}_0) \geq C_k^*(\underline{\mathcal{A}_0}, \mathcal{M}_0), \;\; \forall k$$

so that initial states with higher productivity, ceteris paribus, deliver (weakly) denser baseline networks and higher final consumption.

Lemma (Loose vs Tight Money Baseline)

For any two initial levels money supply such that $\overline{\mathcal{M}_0} > \underline{\mathcal{M}_0}$*:*

$$S_k^*(\mathcal{A}_0, \overline{\mathcal{M}_0}) \supseteq S_k^*(\mathcal{A}_0, \underline{\mathcal{M}_0}) \qquad C_k^*(\mathcal{A}_0, \overline{\mathcal{M}_0}) \geq C_k^*(\mathcal{A}_0, \underline{\mathcal{M}_0}), \;\; \forall k$$

so that initial states with higher money supply, ceteris paribus, deliver (weakly) denser baseline networks and higher final consumption.

MONETARY SHOCKS

Comparative Statics: C and S following $\varepsilon^m \neq 0$

Lemma (Comparative statics after a monetary shock)

A positive monetary shock $\varepsilon^m > 0$, such that $\mathcal{M} > \mathcal{M}_0$, is (weakly) expansionary and makes the equilibrium network (weakly) denser:

$$S_k^*(\mathcal{A}_0, \mathcal{M}) \supseteq S_k^*(\mathcal{M}_0, \mathcal{M}_0) \qquad C_k^*(\mathcal{A}_0, \mathcal{M}) \geq C_k^*(\mathcal{A}_0, \mathcal{M}_0), \ \forall k$$

The opposite holds for a negative monetary shock $\varepsilon^m < 0$, such that $\mathcal{M} < \mathcal{M}_0$.

Definition (Small monetary shock)

Define a monetary shock ε^m to be **small** with respect to the initial state $(\mathcal{A}_0, \mathcal{M}_0)$ if and only if it leaves the equilibrium network unchanged relative to the baseline:

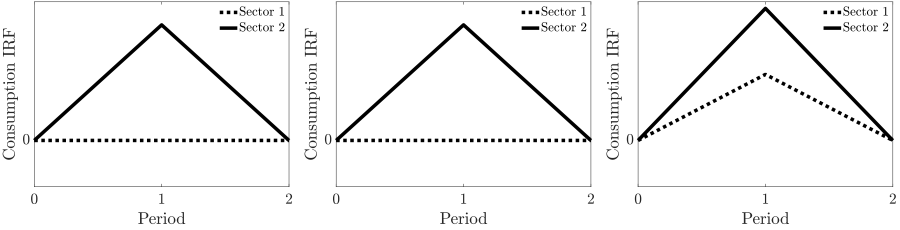
$$S_k^*(\mathcal{A}_0, \mathcal{M}) = S_k^*(\mathcal{A}_0, \mathcal{M}_0), \ \forall k$$

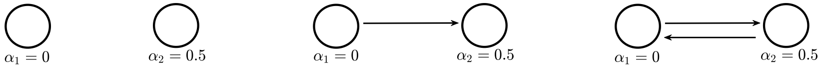
Otherwise, define the monetary shock to be **large** with respect to the initial state $(\mathcal{A}_0, \mathcal{M}_0)$.

Small Monetary Shocks

IRFs to a small monetary expansion across the cycle ε^a

Recession: $\varepsilon^a = 0$ ***Normal***: $\varepsilon^a = 0.65$ ***Expansion***: $\varepsilon^a = 0.8$





Cycle Dependence of the effect of a small $\varepsilon^m \neq 0$

Proposition (IRF in Expansion and Recession)

Consider a monetary shock ε^m that is ***small*** *with respect to both $(\underline{\mathcal{A}_0}, \mathcal{M}_0)$ and $(\overline{\mathcal{A}_0}, \mathcal{M}_0)$, where $\overline{\mathcal{A}_0} \geq \underline{\mathcal{A}_0}$; further let $\hat{\mathbb{C}}_k(\mathcal{A}_0, \mathcal{M}_0; \varepsilon^m)$ be a first order approximation of $\log C_k^*(\mathcal{A}_0, \mathcal{M})$ around $\log C_k^*(\mathcal{A}_0, \mathcal{M}_0)$, then:*

$$\hat{\mathbb{C}}(\overline{\mathcal{A}_0}, \mathcal{M}_0; \varepsilon^m) - \hat{\mathbb{C}}(\underline{\mathcal{A}_0}, \mathcal{M}_0; \varepsilon^m) = \left\{\mathcal{L}(S^*(\overline{\mathcal{A}_0}, \mathcal{M}_0)) - \mathcal{L}(S^*(\underline{\mathcal{A}_0}, \mathcal{M}_0))\right\} A|\varepsilon^{\mathbb{m}}| \geq \mathbb{0}$$

where $\mathcal{L}(S)$ is the Leontief Inverse associated with network S:

$$\mathcal{L}(S) \equiv [I - (1 - A)\Omega(S)]^{-1}$$

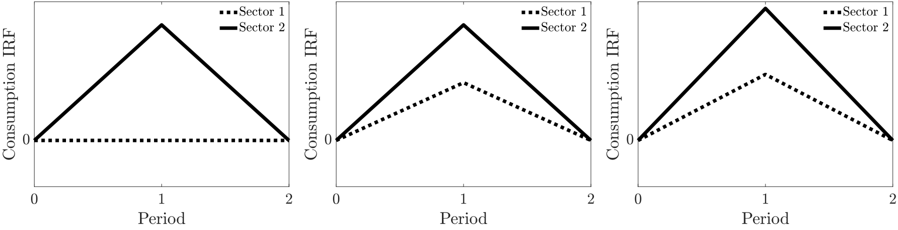
and $\hat{\mathbb{C}} \equiv [\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, ..., \hat{\mathbb{C}}_K]'$, $A = diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $[\Omega(S)]_{kr} = \mathbf{1}_{r \in S_k}\omega_{kr}$, $|\varepsilon^{\mathbb{m}}| \equiv [|\varepsilon^m|, ..., |\varepsilon^m|]'$.
Hence, the magnitude of impulse response of final consumption to a small monetary shock is (weakly) ***procyclical****.*

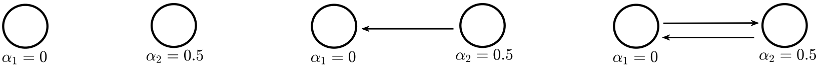
IRFs to a small monetary expansion across initial m_0

Tight money: $m_0 = 0$

Normal money: $m_0 = 4$

Loose money: $m_0 = 8$





Path Dependence of the effect of a small $\varepsilon^m \neq 0$

Proposition (IRF under Loose and Tight money)

*Consider a monetary shock ε^m that is **small** with respect to both $(\mathcal{A}_0, \underline{\mathcal{M}_0})$ and $(\mathcal{A}_0, \overline{\mathcal{M}_0})$, where $\overline{\mathcal{M}_0} \geq \underline{\mathcal{M}_0}$; further let $\hat{\mathbb{C}}_k(\mathcal{A}_0, \mathcal{M}_0; \varepsilon^m)$ be a first order approximation of $\log C_k^*(\mathcal{A}_0, \mathcal{M})$ around $\log C_k^*(\mathcal{A}_0, \mathcal{M}_0)$, then:*

$$\hat{\mathbb{C}}(\mathcal{A}_0, \overline{\mathcal{M}_0}; \varepsilon^m) - \hat{\mathbb{C}}(\mathcal{A}_0, \underline{\mathcal{M}_0}; \varepsilon^m) = \left\{ \tilde{\mathcal{L}}(S^*(\mathcal{A}_0, \overline{\mathcal{M}_0})) - \tilde{\mathcal{L}}(S^*(\mathcal{A}_0, \underline{\mathcal{M}_0})) \right\} |\varepsilon^{\mathbb{m}}| \geq \mathbb{0}$$

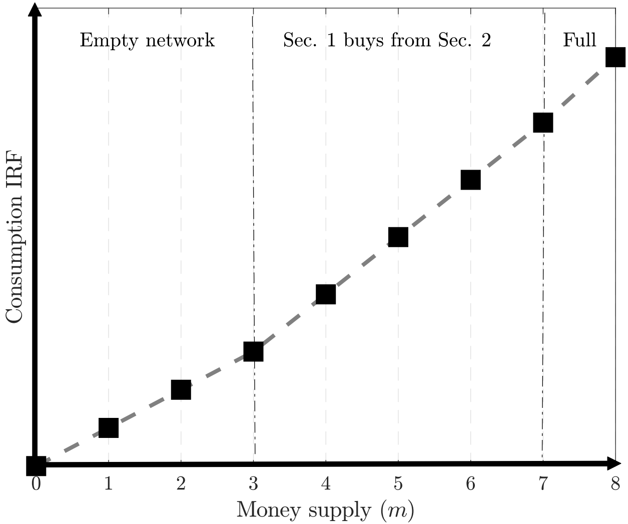
where $\tilde{\mathcal{L}}(S)$ is the adjusted Leontief Inverse associated with network S:

$$\tilde{\mathcal{L}}(S) \equiv [I - (1 - A)\Gamma\Omega(S)]^{-1}[I - (I - A)\Gamma]$$

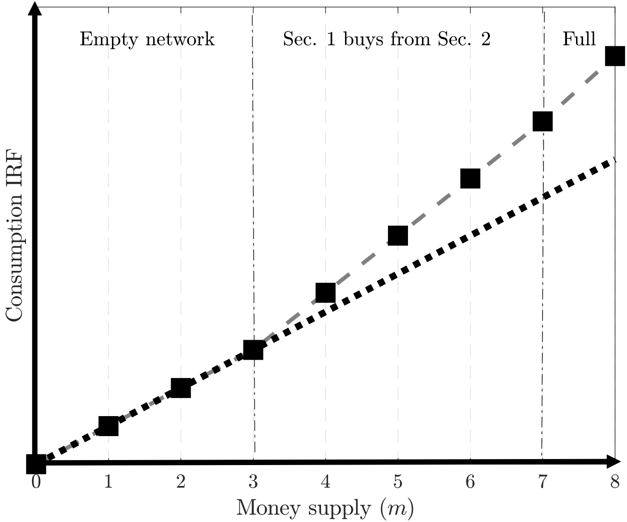
and $\hat{\mathbb{C}} \equiv [\hat{\mathbb{C}}_1, \hat{\mathbb{C}}_2, ..., \hat{\mathbb{C}}_K]'$, $A = diag[\alpha_1, \alpha_2, ..., \alpha_K]'$, $[\Omega(S)]_{kr} = \mathbf{1}_{r \in S_k}\omega_{kr}$, $|\varepsilon^{\mathbb{m}}| \equiv [|\varepsilon^m|, ..., |\varepsilon^m|]'$, $\Gamma = diag[\gamma_1, \gamma_2, ..., \gamma_K]'$, $\gamma_k \equiv \frac{((1+\mu_k)MC_k)^{1-\theta}}{P_{k,0}^{1-\theta} + ((1+\mu_k)MC_k)^{1-\theta}}$, $\forall k$.
*Hence, the magnitude of impulse response of final consumption to a small monetary shock is (weakly) **higher under loose money**.*

Large Monetary Shocks

Large monetary expansions



Large monetary expansions



Time Dependent pricing, **Size Dependent** effects

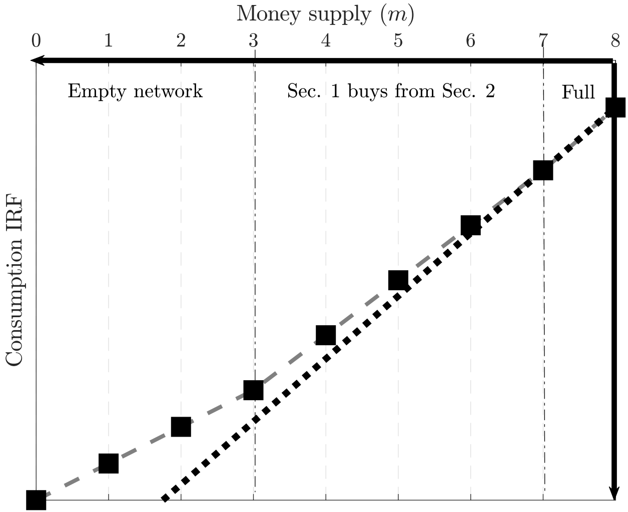
Proposition (Large monetary expansion)

Let $E_+^m > 0$ be a large expansionary monetary shock, and $\varepsilon_+^m > 0$ be a small expansionary monetary shock, both with respect to $(\mathcal{A}_0, \mathcal{M}_0)$; further, denote $S_{E_+} \equiv S^* \left(\mathcal{A}_0, \mathcal{M}_0 \exp(E_+^{\mathcal{M}})\right)$ and $S_0 \equiv S^* \left(\mathcal{A}_0, \mathcal{M}_0 \exp(\varepsilon_+^m)\right) = S^*(\mathcal{A}_0, \mathcal{M}_0)$. It can be shown that:

$$\mathcal{L}(S_0)A(\mathbb{E}_+^m - \varepsilon_+^m) + h.o.t. \leq \hat{\mathbb{C}}^*(\mathcal{A}_0, \mathcal{M}_0; E_+^m) - \hat{\mathbb{C}}^*(\mathcal{A}_0, \mathcal{M}_0; \varepsilon_+^m) \leq \mathcal{L}(S_{E_+}) \quad A(\mathbb{E}_+^m - \varepsilon_+^m) + h.o.t.$$

*Hence, large monetary expansions have a **more than proportional effect on GDP** than small monetary expansions.*

Large monetary contractions



Time Dependent pricing, **Size Dependent** effects

Proposition (Large monetary contraction)

Let $E_{-}^{m} < 0$ be a large contractionary monetary shock, and $\varepsilon_{-}^{m} < 0$ be a small contractionary monetary shock, both with respect to $(\mathcal{A}_0, \mathcal{M}_0)$; further, denote $S_{E_{-}} \equiv S^{*}\left(\mathcal{A}_0, \mathcal{M}_0 \exp(E_{-}^{M})\right)$ and $S_0 \equiv S^{*}\left(\mathcal{A}_0, \mathcal{M}_0 \exp(\varepsilon_{-}^{m})\right) = S^{*}(\mathcal{A}_0, \mathcal{M}_0)$. It can be shown that:

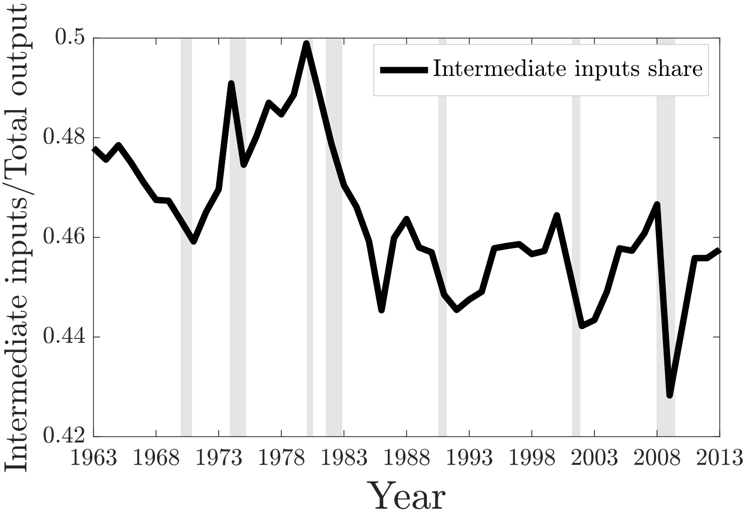
$$\mathcal{L}(S_{E_{-}})A(\varepsilon_{-}^{m} - \mathbb{E}_{-}^{m}) + h.o.t. \leq \hat{\mathbb{C}}^{*}(\mathcal{A}_0, \mathcal{M}_0; E_{-}^{m}) - \hat{\mathbb{C}}^{*}(\mathcal{A}_0, \mathcal{M}_0; \varepsilon_{-}^{m}) \leq \mathcal{L}(S_0) A(\varepsilon_{-}^{m} - \mathbb{E}_{-}^{m}) + h.o.t.$$

*Hence, large monetary contractions have a **less than proportional effect on GDP** than small monetary contractions.*

EMPIRICAL EVIDENCE

Sectoral Data

Intermediates as share of output (BEA, US)



Cyclical fluctuations in intermediates intensity

- Use BEA annual sectoral accounts (KLEMS) to construct sectoral measures of intermediates intensity between 1987-2017 for 65 sectors (Summary level):

$$\delta_{kt} = \frac{\text{Expenditure on Intermediates}_{kt}}{\text{Expenditure on Intermediates}_{kt} + \text{Compensation of Employees}_{kt}}$$

which exactly matches to $\sum_{r \in S_{kt}} \omega_{kr}, \forall k$, in our theoretical framework

- Linear local projection:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H shock_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \qquad H = 0, 1, ..., \overline{H}$$

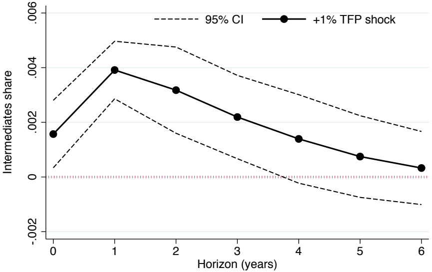
- Non-linear local projection:

$$\delta_{k,t+H} = \alpha_{k,H} + \beta_H^l shock_t + \beta_H^q shock_t^2 + \beta_H^c shock_t^3 + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \ H = 0, 1, ..., \overline{H}$$

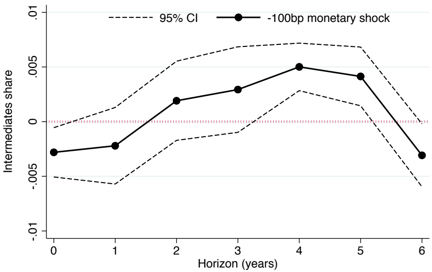
- Use Fernald's TFP shocks and Romer-Romer monetary shocks

Intermediates intensity response: linear local projection

Effect of +1% productivity expansion

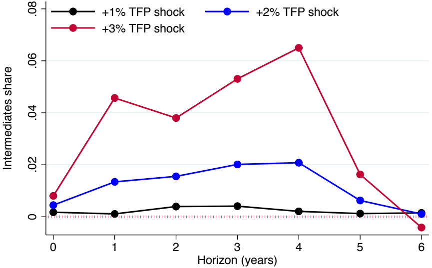


Effect of -100bp monetary easing

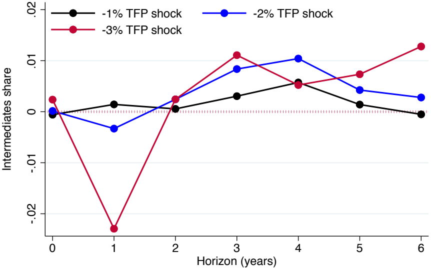


Productivity shocks: non-linear local projection

Productivity expansions

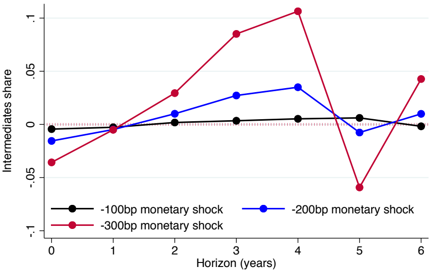


Productivity contractions

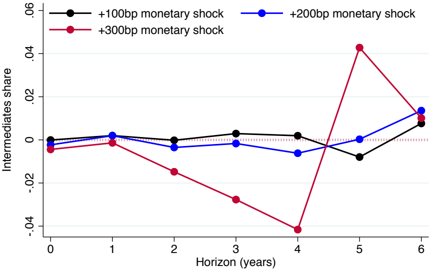


Monetary shocks: non-linear local projection

Monetary expansions

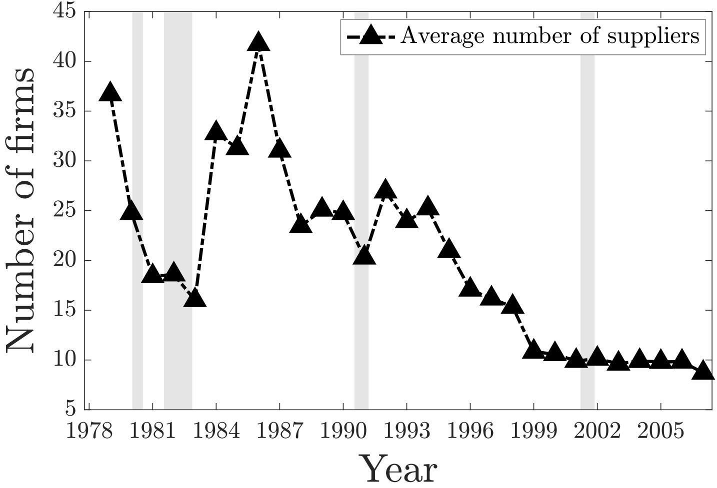


Monetary contractions



Firm-level Data

Number of suppliers (Compustat, US)



Cyclical fluctuations in the number of suppliers

- Measure the number of suppliers at firm level, using data on ”in-degree” computed by Atalay et al. (2011) for US publicly listed firms available in Compustat

- Linear local projection:

$$indeg_{k,t+H} = \alpha_{k,H} + \beta_H shock_t + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \qquad H = 0, 1, ..., \overline{H}$$

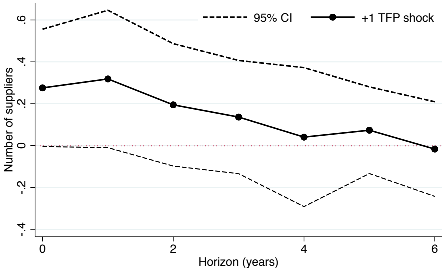
- Non-linear local projection:

$$indeg_{k,t+H} = \alpha_{k,H} + \beta_H^l shock_t + \beta_H^q shock_t^2 + \beta_H^c shock_t^3 + \gamma_H x_{k,t-1} + \varepsilon_{k,t+H}, \; H = 0, 1, ..$$

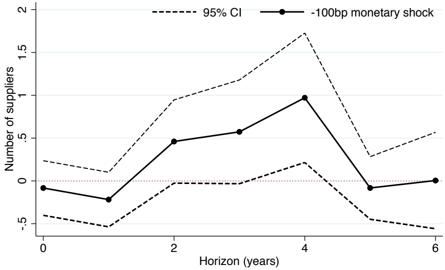
- Use Fernald’s TFP shocks and Romer-Romer monetary shocks

Number of suppliers response: linear local projection

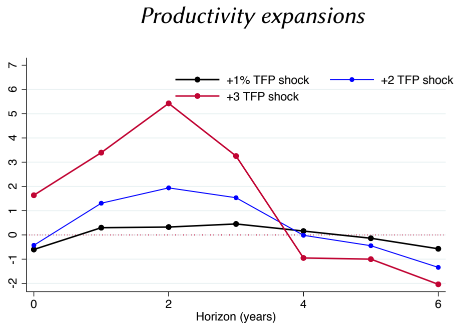
Effect of +1% productivity expansion

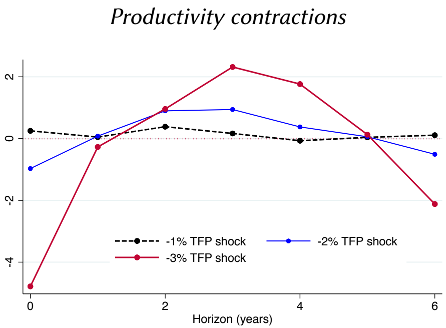


Effect of -100bp monetary easing



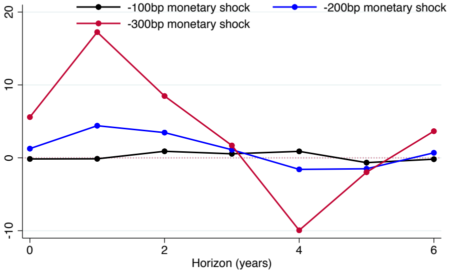
Productivity shocks: non-linear local projection



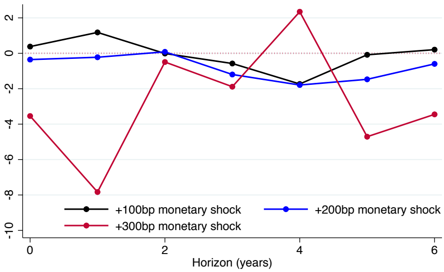


Monetary shocks: non-linear local projection

Monetary expansions



Monetary contractions



Conclusion

- Develop a sticky-price New Keynesian model with endogenous input-output linkages across sectors
- Results rationalize observed non-linearities associated with monetary transmission: cycle dependence, path dependence and size dependence (without using state-dependent pricing)
- Novel empirical evidence in support of the mechanism
- Current work:
  - Quantify the mechanisms in a calibrated multi-sector setting
- Future work: endogenous networks across countries, monetary transmission under varying "openness"

Thank you!

APPENDIX